



Yashwantrao  
Chavan  
Maharashtra  
Open University

**CMP332**

**Quantitative  
Aptitude**

# Quantitative Aptitude

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**Yashwantrao Chavan Maharashtra Open University**

Dnyangangotri, Near Gangapur Dam

Nashik-422222

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**Yashwantrao Chavan Maharashtra Open University**

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Vice-Chancellor: Prof. E. Vayunandan

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**Production**

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**Course Objectives:**

- To enhance the problem solving skills, to improve the basic mathematical skills and to help students who are preparing for any type of competitive examinations.
- Communication Goal: Students will be able to interpret and communicate quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.
- Problem Solving Goal: Students will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.
- Reasoning Goal: Students will be able to reason, model, and draw conclusions or make decisions with mathematical, statistical, and quantitative information.
- Evaluation Goal: Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.
- Technology Goal: Students will be able to use appropriate technology in a given context.

**Learning Outcomes:**

On successful completion of the course the students will be able to:

- Understand the basic concepts of QUANTITATIVE ABILITY
- Understand the basic concepts of LOGICAL REASONING Skills
- Acquire satisfactory competency in use of VERBAL REASONING
- Solve campus placements aptitude papers covering Quantitative Ability, Logical Reasoning and Verbal Ability
- Solve real-life problems requiring interpretation and comparison of complex numeric summaries which extend beyond simple measures of centre.
- Distinguish between proportional and non proportional situations and, when appropriate, apply proportional reasoning.
- Students will apply probabilistic reasoning to draw conclusions, to make decisions, and to evaluate outcomes of decisions.
- Students will draw conclusions or make decisions and communicate their rationale based on understanding, analysis, and critique of self-created or reported statistical information and statistical summaries.
- Compete in various competitive exams like CAT, CMAT, GATE, GRE, GATE, UPSC, GPSC etc.

Unit No. and Name	Details	Counseling Sessions	Weightage
Unit 1 : Number System Basics	<ul style="list-style-type: none"><li>• Number system</li><li>• Divisibility</li><li>• Factors</li><li>• HCF and LCM</li></ul>		
Unit 2 : Averages and Problems on Ages	<ul style="list-style-type: none"><li>• Arithmetic Mean</li><li>• Geometric Mean</li><li>• Harmonic Mean</li><li>• Mean, Median and Mode</li></ul>		

Unit 3 : Percentages, Profit and Loss	<ul style="list-style-type: none"> <li>• Basic Concepts of Percentages</li> <li>• Successive Discounts</li> <li>• Percentage Error</li> <li>• Increase and Decrease in Percentage</li> <li>• Basics of Profit and Loss</li> <li>• Formulas to calculate Profit &amp; Loss</li> <li>• Profit Percentage</li> </ul>		
Unit 4 : Ratio and Proportion, Partnership, Mixtures and Allegations, Simple Interest and Compound Interest	<ul style="list-style-type: none"> <li>• Basics of Ratio and Proportion</li> <li>• Continued Proportion</li> <li>• Comparison of Ratios</li> <li>• Variations</li> <li>• Understanding of Simple and Compound Interest</li> <li>• EMI Calculation</li> </ul>		
Unit 5 : Work and Time and Geometry and Trigonometry	<ul style="list-style-type: none"> <li>• Basics on Work</li> <li>• Time and Work Formula and Application</li> <li>• Pipes and Cisterns</li> <li>• Chain Rule</li> <li>• Basics on Geometry and Trigonometry</li> <li>• <u>Types, methodologies, and terminologies of geometry.</u></li> <li>• Formulas of functions of Trigonometry</li> </ul>		
Unit 6 : Speed, Distance and Time	<ul style="list-style-type: none"> <li>• Basics of Speed Distance and Time</li> <li>• Average Speed</li> <li>• Relative Speed</li> <li>• Problems on Trains Boats and Streams</li> <li>• Problems on Boats</li> </ul>		
Unit 7 : Permutation and Combination, Probability	<ul style="list-style-type: none"> <li>• Basics of Permutation and Combination</li> <li>• Representation of Permutation and Combination</li> <li>• Problems on Permutation and Combination</li> <li>• Basics of Probability</li> <li>• Variables and Distributions</li> <li>• Problems on Permutation, Combination and Probability</li> </ul>		
Unit 8 : Introduction to Tables and Graph	<ul style="list-style-type: none"> <li>• Vertical Bar Charts.</li> <li>• Histogram.</li> <li>• Horizontal Bar.</li> <li>• Pie Charts.</li> <li>• Line Charts.</li> <li>• And other Graphs and Tables.</li> </ul>		

#### Reference Books:

1. Quantitative aptitude by R.S Agarwal
2. Arihant Publications - Fast Track Objective Arithmetic
3. R.D. Sharma - Mathematics Class 11th and 12th
4. Sarvesh K. Verma- Quantitative Aptitude

Note: This Study material is still under development/editing process and is being made available for the sole purpose of reference. Final edited copies will be made available once ready.

## UNIT - 1 NUMBER SYSTEM

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### Learning Objective

**After going through this unit, you will be able to understand:**

- What is the basic idea of Natural Numbers, Whole Numbers, Prime nos etc.
  - Moderate questions on Divisibility, Remainders, Factors, HCF and LCM
  - Understand unit place, tenth place
  - Able to frame approaches on questions on Number System
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### 1.1 Theoretical Framework of Number System

#### Introduction to Number System

The *number system* mainly into classified into 10 types.

1. Complex Numbers
2. Imaginary Numbers
3. Real Numbers
4. Rational Numbers
5. Irrational Numbers
6. Integers
7. Whole Numbers
8. Natural Numbers
9. Prime Numbers
10. Perfect Numbers

### **Complex Numbers:**

Complex numbers are formed by the addition of a real number and an imaginary number, the general form of which is  $a + bi$  where  $i = \sqrt{-1}$  = the imaginary number and  $a$  and  $b$  are real numbers. The " $a$ " is said to be the real part of the complex number and  $b$  the imaginary part. *We don't come across at Complex numbers and Imaginary numbers in Aptitude.*

### **Imaginary Numbers**

a number that is expressed in terms of the square root of a negative number usually the square root of  $-1$ , represented by  $i$  or  $j$  ). When imaginary numbers are squared, they yield a negative result.

### **Real Numbers**

The real numbers is the set of numbers containing all of the rational numbers and all of the irrational numbers. The real numbers are "all the numbers" on the number line.

### **Rational Numbers**

The **rational numbers** are those numbers which can be expressed as a ratio between two integers. For example, the fractions  $1/13$ ,  $23/1$ . All the integers are included in the rational numbers, since any integer can be written as a ratio of integer divided by 1, e.g.  $23/1$ .

### **Irrational Numbers**

An **irrational number** is a number that cannot be written as a ratio or fraction). In decimal form, it never ends or repeats. e.g. under root of 2, the answer is 1.41421356237309.....

### **Integers**

The **integers** are the set of real numbers consisting of the natural numbers, their additive inverses and zero. ...-3, -2, -1, 0, 1, 2, 3...)

### **Whole Numbers**

A simple definition would be all the natural numbers plus the zero. 0,1,2,3,4,5....)

### **Natural Numbers**

the set of numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,....., that we see and use every day. The natural numbers are often referred to as the counting numbers and the positive integers.

### **Prime Numbers**

A number which is not exactly divisible by any number except itself and unity is called a prime number. The number 2 is the least prime number and the only even prime number. Unity is neither a prime nor a composite number.

The largest prime number known so far is  $2^{19937} - 1$ , which is of 6002 digits.

### **Relatively Prime Numbers**

The numbers p and q are called relatively prime if HCF of p and q is 1. They are also called co-prime numbers.

### **Test for Prime Numbers**

For testing any number for being prime or not follow the below mentioned steps: Method I:

1. Take the nearest integer larger than the approximate square root of that number, suppose it is p.
2. Test the divisibility of the given number by every prime number less than p.
3. If the number is not divisible by any one of them, then it is a prime number, otherwise, it is a composite number.

Method II: If you can express the given number in the form of  $6k+1$  or  $6k-1$ , then the number is a prime number.

### **Prime Factors**

Any composite number (whole number) can be written as a product of factors which are prime numbers. This is known as Prime Factorization or Canonical Form of a number.

### **Perfect Number**

A number is said to be a perfect number if sum of its factors excluding the number itself but including 1 is equal to the number. Eg. 6.

A number of the form  $2^n - 1$ , where 'n' is a positive integer such that  $2^n - 1$  is a prime number.

#### Important Property of Perfect Numbers

The sum of the reciprocals of all the divisors including the number itself and unity) of a perfect number is equal to two.

### **1.3 Divisibility Rules/Tests**

#### Tests for Divisibility

A number (Dividend) is said to be divisible by another number called Divisor) when the quotient is a natural number and remainder is zero.

The relation so obtained can be given by  $\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$ .

1. *Divisibility by 2:* A number is divisible by 2 if its unit digit is even.

2. *Divisibility by 3:* A number is divisible by 3 when the sum of its digits is divisible by 3.

3. *Divisibility by 4:* A number is divisible by 4 when the number formed by its two extreme right digits i.e. tens digit and unit digit is divisible by 4.



4. *Divisibility by 5:* A number is divisible by 5 when its unit digit is either 5 or 0.

5. *Divisibility by 6:* A number is divisible by 6 when it is divisible by 2 and 3.

6. *Divisibility by 7:*

a. Method A – Write the given number then

i. Write the number leaving its extreme right digit and subtract from it the product of extreme right digit and 2. The balance obtained is a new number.

ii. Repeat step i) until you can be sure to say whether the balance obtained is divisible by 7 or not.

b. Method B – When the number has more than 3 digits, follow these steps:

i. Group the number in three from unit digit.

ii. Add the odd groups and even groups separately.

iii. The difference of the odd groups and even groups should be divisible by 7.

7. *Divisibility by 8:* A number is divisible by 8 when the number formed by its three extreme right digits i.e. hundreds digit, tens digit and unit digit is divisible by 8.

8. *Divisibility by 9:* A number is divisible by 9 when the sum of its digits is divisible by 9.

9. *Divisibility by 10:* A number is divisible by 10 when it has zero 0) at its unit's place.

10. *Divisibility by 11:* A number is divisible by 11 when the difference between the sum of the digits at odd places and the sum of the digits at even places is either zero 0) or divisible by 11.

11. *Divisibility by 12:* A number is divisible by 12 when it is divisible by 3 and 4.

*12. Divisibility by 13:*

a. Method A – Write the given number then

- i. Write the number leaving its extreme right digit and add to it the product of the extreme right digit and 4. The sum obtained is a new number.
- ii. Repeat step i) until you can be sure to say whether the new number obtained is divisible by 13 or not.

b. Method B – When the number has more than 3 digits follow these steps

- i. Group the number in three from unit digit.
- ii. Add the odd groups and even groups separately.
- iii. The difference of the odd and even groups should be divisible by 13.

*13. Divisibility by 14:* A number is divisible by 14 when it is divisible by both 2 and 7.

*14. Divisibility by 15:* A number is divisible by 15 when it is divisible by 3 and 5.

*15. Divisibility by 16:* A number is divisible by 16 when the number formed by its four extreme right digits i.e. thousands digit, hundreds digit, tens digit and unit digit is divisible by 16.

*16. Divisibility by 17:* Write the number and follow these steps

- i. Write the number leaving its extreme right digit and subtract from it the product of extreme right digit and 5. The balance obtained is the new number.
- ii. Repeat step i) until you can be sure to say whether the balance obtained is divisible 17 or not.

*17. Divisibility by 18:* A number is divisible by 18 when it is divisible by 2 and 9.

*18. Divisibility by 19:* Write the number and follow these steps.

- i. Write the number leaving its extreme right digit and add to it the product of extreme right digit and 2. The sum obtained is a new number.
- ii. Repeat step i) until you can be sure to say whether the new number obtained is divisible by 19 or not.

*19. Divisibility by  $5^n$ :* A number is divisible by  $5^n$  when the number formed by its 'n' extreme right digits is divisible by  $5^n$  or these digits are zeroes.

## 1.2 SOLVED EXAMPLES - Divisibility

**Question 1:**  $60x6 \times 111 = 666666$ , find  $x$ .

- a. 0                      b. 1                      c. 2                      d. 3

**Explanation:**

→ We will get the answer by dividing 666666 by 111, the answer is 6006.

**Question 2:** If 23334 $x$  is divisible by 5, Find number of possible values of  $x$ .

- a. 1                      b. 2                      c. 3                      d. 4

**Explanation:**

As the number is divisible by 5, the units place would be either 0 or 5, hence the number of values are 2, Option (b) is correct.

**Question 3:** 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. What is the first part?

- a. 13                      b. 15                      c. 19                      d. none of these

**Explanation:**

- One way to solve would be to use options and check if any of the options match the criteria, also you can solve it using algebra, by creating 2 equations  
→ The first equation would be  $x + y = 24$   
→ The second equation would be  $7x + 5y = 146$ , solving both the equations we get the answer as  $x = 13$  and  $y = 11$ ,

**Question 4:**  $\frac{4}{5}$  of a certain number is 64. Find half of that number.

- a. 32                      b. 40                      c. 80                      d. 16

**Explanation:**

- Forming equation would be  $\frac{4x}{5} = 64$ , solving this, the number would be 80, but the question is finding the half of this number which is 40, which is the right answer.

**Question 5:**  $\frac{4}{5}$  of a number exceeds its  $\frac{2}{3}$  by 8. What is the number?

- a. 30                      b. 60                      c. 90                      d. 120

**Explanation:**

→ Again we can check all the options or we can create two equations;

→  $(4x/5) - (2x/3) = 8$ , solving this we get the answer as 60.

## **1.3 LCM and HCF**

### ***1.3.1 Least Common Multiple LCM)***

Least common multiple of two or more numbers is the least number which is divisible by each of these numbers without leaving a remainder. It is also known as Lowest Common Dividend.

Methods of Finding LCM of Given Numbers

There are two methods which are usually used to find LCM of two or more numbers.

#### ***1. Prime Factorization Method:***

Resolve each one of the given numbers into prime factors. The LCM is the product of the highest power of all prime factors.

#### ***2. Division Method:***

In this method, we divide the given numbers by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. We keep on repeating this process until no two numbers have a common factor. The product of the divisors and the remaining numbers is LCM of the given numbers.

### ***1.3.2 Highest Common Factor (HCF)***

Highest Common Factor of two or more given numbers is the largest common factor. It is also known as the Greatest Common Factor (GCF) or Greatest Common Divisor (GCD) or Greatest Common Measure (GCM).

#### **Methods of Finding HCF of Given Numbers**

There are two methods that are usually used to find the HCF of given numbers.

##### ***1. Prime Factorization Method:***

In this method, we write prime factors of each of the given numbers in the exponential form. HCF is the product of the common prime factors with least powers.

##### ***2. Long Division Method:***

In this method, we divide the largest number by smaller number and get a remainder. Then we divide the first divisor by the remainder getting a new remainder and continue this process till the last number is zero. The last divisor in this process is the HCF of the given two numbers.

In order to find the HCF of three or more numbers, follow these steps:

- i. First find the HCF of any two numbers.
- ii. Then find the HCF of the third number and the HCF obtained above.
- iii. HCF obtained in step ii) is the required HCF of the given three numbers.

##### ***Euclid's Algorithm***

This is used for large numbers. In this method when  $a > b$ ,  $\text{HCF } a, b = \text{HCF } b, a - b$ .

### ***1.3.3 HCF and LCM of Fractions***

i)  $\text{HCF of Fractions} = \text{HCF of Numerators} / \text{LCM of Denominators}$  ii)  $\text{LCM of Fractions} = \text{LCM of Numerators} / \text{HCF of Denominators}$

### ***Product of Two Numbers***

If two numbers are A and B, then  $\text{HCF}(A, B) \times \text{LCM}(A, B) = A \times B$ . This formula is applicable for two numbers only.

*Please note:*

1. For any two digit numbers, HCF is a factor of LCM.
2. Any number which divides each of the two numbers also divides their sum, difference, and also difference of their multiples.

### ***1.3.4 HCF and LCM of Decimals***

In given numbers make the same number of places of decimals by annexing zeroes in a number if necessary. Consider these numbers without decimal points, find HCF or LCM as the case may be. Now, in the result, mark off as many decimal places as there are in each of the numbers.

Important Results

1. Lowest number that is divisible by A, B, C leaving same remainder 'r' in each case is  $\text{LCM}(A, B, C) + r$ .
2. Greatest number that will divide A, B, C leaving remainders  $r_1, r_2$ , and  $r_3$  respectively is  $\text{HCF of } (A - r_1), (B - r_2), \text{ and } (C - r_3)$ .
3. If a, b and c are such that  $a < b < c$  and if H and L are HCF and LCM of a, b and c respectively, then  $1 \leq H \leq a$  and  $a \leq L \leq abc$ .
4. Let H be HCF of a, b and c, then  $a - b$ ,  $b - c$ , and  $c - a$  is also divisible by H.

## 1.4 SOLVED EXAMPLES - HCF

**Question 1:** Find HCF of 70 and 90.

- a. 5                      b. 10                      c. 15                      d. 1

**Explanation:**

→ Using divisibility on both the numbers, we get the common factors:

→  $70 = 2 \times 5 \times 7$  and for  $90 = 2 \times 3 \times 3 \times 5$ , the HCF is  $2 \times 5 = 10$ .

**Question 2:** What is the LCM of 25, 30, 35 and 40?

- a. 2000                      b. 4200                      c. 2800                      d. 3588

**Explanation:** Solving this using divisibility,

→  $25 = 5 \times 5$ ;  $30 = 2 \times 3 \times 5$ ;  $35 = 5 \times 7$  and  $40 = 2 \times 2 \times 2 \times 5$ , taking the common multiples which are 5 common in all),  $\times 2$  common in 2)  $\times 5 \times 3 \times 7 \times 2 \times 2$  uncommon) multiplying ll we get the answer as 4200.

**Question 3:** What is the LCM of  $2/5$ ,  $3/10$  and  $6/25$ ?

- a.  $2/3$                       b.  $3/8$                       c.  $6/5$                       d.  $11/50$

**Explanation :**

→ To find LCM and HCF of  $a/b$ . and  $c/d$ . the generalized formula will be:

→  $H.C.F = H.C.F \text{ of numerators} / L.C.M \text{ of denominators}$

→  $L.C.M = L.C.M \text{ of numerators} / H.C.F \text{ of denominators}$

→ Hence, solving using this, the answer is  $6/5$ .

**Question 4:** What is the LCM of 1.2, 0.24 and 6?

- a. 2                      b. 4                      c. 6                      d. 8

**Explanation -**

→ 1.2 can be written as  $120/100$ , 0.24 can be written as  $24/100$  and 6 can be written as  $600/100$ , we need to make the denominator common, hence multiplying the denominator to get it as 100.

→ Solving this using the above formula, the answer would be  $600/100$  hence 6



→ This can be solved by multiplied all the numbers by 100 also. So, LCM of 120, 24 and 600) and then divide it by 100.

**Question 5: Find HCF of 6.16 and 13.**

- a. 0.02      b. 0.04      c. 1      d. none of these

**Explanation:**

→ You can go with method used in Question 4 also.

→ Method 2: Factorise 6.16 \*100) and 13\*100).

→  $616 = 2^2 * 154$

→  $= 2^3 * 77$   
 $= 2^3 * 7 * 11$

→  $1300 = 13 * 2^2 * 5^2$

→ HCF  $= 2^2$  i.e. 4

*But as we had multiplied by 100 earlier, we have to divide the actual answer by 100 now)*

So, actual answer  $4/100 = 0.04$ . Cheers!

**Question 6: Which greatest possible length can be used to measure exactly 1meter 65 cm, 1meter 95 cm and 75 cm?**

- a. 45cm      b. 15cm      c. 125cm      d. 55cm

**Explanation:**

→ 1 meter 65 cm is equal to 165 cm, 1 meter 95 cms is equal to 195 cm and 75 cms. We need to find out the HCF to find out the greatest possible length.

→ Solving this, we get the answer as 15 cms.

**Question 7: An oil merchant has three different kinds of oil: 435 liters, 493 liters and 551 liters. Find the least number of casks of equal size required to store all the oil without mixing.**

- a. 29      b. 41      c. 51      d. 57

**Explanation:**

→ Least number of cask  $= 435 / \text{HCF } 435, 493, 551) + 493 / \text{HCF } 435, 493, 551) + 551 / \text{HCF } 435, 493, 551).$

→  $435 = 3 * 5 * 29$

→  $493 = 17 * 29$

→  $551 = 19 * 29$

→  $HCF = 29$ .

→ So, Least no of casks =  $435/29 + 493/29 + 551/29 = 15 + 17 + 19 = 51$  Yahoo!)

**Question 8:** *The traffic lights at a T-junction change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8:20:00 hours, then they will again change simultaneously at:*

- a. 8:27:12 hrs      b. 8:27:24 hrs      c. 8:27:36 hrs      d. 8:27:48 hrs

**Explanation:**

→ We need to find out when the lights will blink again together hence the LCM of the time taken by the lights to blink to find out the common time for them to blink.

→ So, LCM of 48, 72, 108 is 432, hence after 432 secs they will blink again, now we have to convert 432 secs into min, so 7 mins and 12 secs after 8:20:00, so the answer would be 8:27:12 hrs.

**Question 9:** *A room is 4 meters 37 cm long and 3 meters 23 cm broad. It is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose.*

- a. 19      b. 391      c. 441      d. None of these

**Explanation:** Refer Question no. 7:

→  $HCF$  of 437, 323

→  $437 = 19 * 23$       *Catch:* to identify factors do  $437-323$  i.e.114. Now factors of 437 and 323 has to be multiple of 114 and hence we got 19)

→  $323 = 17 * 19$

→  $HCF = 19$

→ Minimum least) no. of square slabs =  $437/19 + 323/19 = 40$

→ Hence, none of these is right choice!

**Question 10:** *Five bells begin to toll together and toll at intervals of 36, 45, 72, 81 and 108 seconds respectively. After what interval of time will they keep on tolling together?*

- a. 1620 sec      b. 3240 sec      c. 3200 sec      d. 3080 sec

***Explanation:***

- This is again another type of LCM question, we need to find out the LCM for 36, 45, 72, 81 and 108
- Solving this we get:  $36 = 2 \times 2 \times 3 \times 3$
- $45 = 3 \times 3 \times 5$
- $72 = 2 \times 2 \times 2 \times 3 \times 3$
- $81 = 3 \times 3 \times 3 \times 3$
- $108 = 2 \times 2 \times 3 \times 3 \times 3$
- LCM of more than 2 number is derived by taking out the common number in all the 5 numbers, multiplying it with common in 4 numbers, multiplying it with 3 then with 2 common, then all the uncommon numbers.
- Solving this we get -  $2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 3 \times 5$  which is equal to 3240 secs, which is our answer.

## 1.5 CONCEPT: CYCLICITY OF NUMBERS

Number System has its application in almost every other topic in mathematics. This very much defines the importance of this topic. The Number System mainly includes further sub-topics like HCF and LCM, unit digit, factors, cyclicity, factorials, Euler number, digital root, etc.

To understand the concept of unit digit, we must know the concept of cyclicity. This concept is mainly about the unit digit of a number and its repetitive pattern on being divided by a certain number

The concept of unit digit can be learned by figuring out the unit digits of all the single digit numbers from 0 - 9 when raised to certain powers.

These numbers can be broadly classified into three categories for this purpose:

1. *Digits 0, 1, 5 & 6*: When we observe the behavior of these digits, they all have the same unit's digit as the number itself when raised to any power, i.e.  $0^n = 0$ ,  $1^n = 1$ ,  $5^n = 5$ ,  $6^n = 6$ . Let's apply this concept to the following example.

Example: Find the unit digit of following numbers:

- $185^{563}$

Answer= 5

- $271^{6987}$

Answer= 1

- $156^{25369}$

Answer= 6

- $190^{654789321}$

Answer= 0

2. *Digits 4 & 9*: Both these numbers have a cyclicity of only two different digits as their unit's digit.

Let us take a look at how the powers of 4 operate:  $4^1 = 4$ ,

$$4^2 = 16,$$

$$4^3 = 64, \text{ and so on.}$$

Hence, the power cycle of 4 contains only 2 numbers 4 & 6, which appear in case of odd and even powers respectively. This means, 4 raised to odd power, unit place would be 4 and when raised to even power, unit place would be 6.

Likewise, the powers of 9 operate as follows:

$$9^1 = 9,$$

$$9^2 = 81,$$

$$9^3 = 729, \text{ and so on.}$$

Hence, the power cycle of 9 also contains only 2 numbers 9 & 1, which appear in case of odd and even powers respectively.

So, broadly these can be remembered in even and odd only, i.e.  $4^{\text{odd}} = 4$  and  $4^{\text{even}} = 6$ . Likewise,  $9^{\text{odd}} = 9$  and  $9^{\text{even}} = 1$ .

**Example: Find the unit digit of following numbers:**

- $189^{562589743}$

Answer = 9 since power is odd.

- $279^{698745832}$

Answer = 1 since power is even)

- $154^{258741369}$

Answer = 4 since power is odd.

- $194^{65478932}$

Answer = 6 since power is even)

3. *Digits 2, 3, 7 & 8:* These numbers have a power cycle of 4 different numbers.

$2^1 = 2$ ,  $2^2 = 4$ ,  $2^3 = 8$  &  $2^4 = 16$  and after that it starts repeating.

So, the cyclicity of 2 has 4 different numbers 2, 4, 8, 6.

$3^1 = 3$ ,  $3^2 = 9$ ,  $3^3 = 27$  &  $3^4 = 81$  and after that it starts repeating.

So, the cyclicity of 3 has 4 different numbers 3, 9, 7, 1.

7 and 8 follow a similar logic.

So these four digits i.e. 2, 3, 7 and 8 have a unit digit cyclicity of four steps.

### *Cyclicity Table*

The concepts discussed above are summarized in the given table.

<b>Number</b>	<b>Cyclicity</b>	<b>Power Cycle</b>
1	1	1
2	4	2, 4, 8, 6
3	4	3, 9, 7, 1
4	2	4, 6
5	1	5
6	1	6
7	4	7, 9, 3, 1
8	4	8, 4, 2, 6
9	2	9, 1
10	1	0

## 1.6 SOLVED EXAMPLES - CYCLICITY

**Question 1: Find the Unit digit of  $287^{562581}$**

**Explanation:**

- Step 1: We know that the cyclicity of 7 is 4.
- Step 2: Divide the power 562581 by 4.
- By doing that, we get a remainder = 1 Catch: Divide only last 2 digits - Divisibility!)
- Step 3: 1st power in the power cycle of 7 is 7.
- Hence, the answer is 7.

**Question 2: Find the Unit digit of  $13445 * 54336$**

**Explanation:**

- Cyclicity of 5 & 6 is 1. Since  $5*6=30$ , the unit digit of given expression is 0.

**Question 3: Find the last digit of  $3119^{234} * 234^{3119}$**

**Explanation:**

- We only bother of last digit of the number. So, our question gets reduce to  $9^{234} * 4^{3119}$ . As we know,  $9^{\text{even power}}$  - unit place is 1 and  $4^{\text{odd power}}$ , unit place is 4.
- So, Final answer:  $1*4 = 4$ .

**Question 4: Find unit digit of  $1^{10} * 2^{10} * 3^{10} * \dots 99^{10}$ .      *Interesting one!***

**Explanation:**

- As we are multiplying unit digits of all expressions, there would be numbers like  $10^{10}$ ,  $20^{10}$  and so on, whose unit place will be 0. So, unit place of this expression will be 0.

**Question 5: Find the unit place of  $13^{41}$ .**

**Explanation:** Again, we are concerned with unit place i.e. 3. Cyclicity of powers of 3 is 3, 9, 7, 1 i.e.4). So, we divide power 41 by 4. Remainder is 1 and so, answer will be the first term of the cycle which is 3. Yep!

**Question 6: Find the last digit of  $222^{888} + 888^{222}$**

**Explanation:**

- Expression reduced to  $2^{888} + 8^{222}$
- Cyclicity of 2 is 4. So, divide 888 by 4. Remainder is 0. Last term of cycle as remainder is 0) which is 6.
- Cyclicity of 8 is 4. So, divide 222 by 4. Remainder is 2. So, 2nd term of cycle of 8 is 4.
- Final unit place  $\dots 6 + \dots 4 = \dots 0$  Hurray)

**Question 7. Find the unit digit of the expression:  $888^{888!} + 222^{222!} + 333^{333!} + 777^{777!}$ .**

- a. 3                      b. 4                      c. 5                      d. 6

**Explanation:**

- The example seems quite difficult one. But once you are clear with basic fundamentals and if not, scroll up the pages), things will be pretty easy.
- Now the expression gets reduce to:  $8^{888!} + 2^{222!} + 3^{333!} + 7^{777!}$
- Cyclicity of powers of 8, 2, 3 and 7 are 4 only. All powers given in factorial are necessarily divided by 4. So, all the given powers when divided by 4, the remainder will be 0 only. So, we are concerned with the last term of cyclicity of the given numbers:
- Hence,  $6 + 6 + 1 + 1 = 4$ . Final answer is 4.

**Question 8: Find the last digit of expression:  $454^{33} - 42^{25}$**

**Explanation:**

- Last place of  $4^{33}$  will be 4 only, whereas last place  $2^{25}$  will be 2.
- So, unit place of this subtraction will be  $4 - 2 = 2$ .
- Example 9: Find the last place of  $131^{32} * 32^{131}$
- Solution: We have already discussed about this type earlier.
- The expression gets reduced to  $1^{32} * 2^{131}$
- $1 * 8 = 8$

## 1.7 CONCEPT OF REMAINDER

Remainders is a very crucial concept since numerous questions from Quantitative Aptitude section requires the concepts of remainder to solve them. Most of the candidates have already



studied this concept in their elementary schools and can solve the related questions. Here is a lesson on Remainders to help candidates revise the topic in an efficient way.

### **What is Remainder?**

Supposing a number “N” is divided by another number “x”; if the quotient obtained is “Q” and the remainder obtained is “R”, then the number can be expressed as  $N = Qx + R$

For example, suppose 8 is divided by 3.

In this case,  $N=8$ ,  $x=3$ .  $3 \times 2 = 6$ , which is 2 less than 8. hence  $Q=2$  and  $R=8-6=2$  Hence,  $8=2 \times 3 + 2$ .

### *Basic Remainder Theorem*

The basic remainder theorem is based on the product of individual remainders.

If R is the remainder of an expression  $p \times q \times r / X$ , and pR, qR and rR are the remainders when p, q and r are respectively divided by X, then it can be said that  $pR \times qR \times rR / X$ , will give the same remainder as given by  $p \times q \times r / X$ .

Let's understand this with the help of some examples.

**Example 1: Find the remainder when  $361 \times 363$  is divided by 12.**

**Solution:**

- Take the product of individual remainders, i.e.  $361/12 | R = 1$  and  $363/12 | R = 3$
- Find the remainder when you divide that product by the number  $361 \times 363 / 12 | R = 1 \times 3 / 12 | R$ . answer = 3
- This is Basic Remainder theorem put across in Numbers.

**Example 2: Find the remainder when  $10^6$  is divided into 7 i.e.  $10^6/7$  R.**

**Solution:**

- $10^6 = 10^3 \times 10^3$
- Thus  $(10^6/7)R = 10^3/7 \times 10^3/7 | R = (6 \times 6)/7 | R = (36/7)R = 1$ .
- So the remainder is 1.

*“Remainder when the product of some numbers is divided by the requisite number is the product of individual remainders of the numbers”*– This is Basic Remainder Theorem put across in words.

### ***Concept of Negative Remainder***

The remainder obtained by division of a number N by a divisor X can be expressed in two ways as “R” and “X-R”

*For example*,  $10/11$  remainder is +10 itself. It can also be written as  $10-11 = -1$  Similarly,  $32/10$  remainder is +2 or -8

Let's express the solution for questions 31 above, in another way- based on the concept of negative remainder. Thus  $106/7$  R =  $103/7$  x  $103/7$  R =  $-1 * -1/7$  R =  $1/7$  R = 1.

Let's see why this happens:

If the numbers  $N_1, N_2, N_3$  give remainders of  $R_1, R_2, R_3$  with quotients  $Q_1, Q_2, Q_3$  when divided by a common divisor D

$$N_1 = DQ_1 + R_1 \quad N_2 = DQ_2 + R_2 \quad N_3 = DQ_3 + R_3$$

$$\text{Multiplying} = N_1 \times N_2 \times N_3$$

$$= (DQ_1 + R_1) \times (DQ_2 + R_2) \times (DQ_3 + R_3) = (\text{some number}) + (R_1 \times R_2 \times R_3) = \text{first part is divisible by D, hence you need to check for the individual remainders only.}$$

## 1.8 SOLVED EXAMPLES - REMAINDERS

**Question 1:** Find the remainder when  $10^{20^{30}}$  is divided by 3

**Explanation:**

- $10^{20^{30}}$  is actually  $10^{600 \cdot 20 \cdot 30}$ : laws of indices)
- The remainder of  $10^1$  divided by 3 is 1
- The remainder of  $10^2$  divided by 3 is 1 and so on and so forth, so any power of 10 divided by 3 will always give you a remainder of 1.

**Question 2:** Find the remainder when  $47^{45}$  is divided by 4.

**Explanation:**

- When  $47^1$  is divided by 4 we get the remainder as 3, when we divide  $47^2 \cdot 2209 / 4$ ) with 4 we get the remainder as 1, when we divide  $47^3 \cdot 103823 / 4$ ) with 4 we get the remainder as 3 and so on and so forth, hence by applying logic if the power of 47 is odd we get the remainder as 1 and if the power of 47 is even, we get the remainder as 3, in the above example the power of 47 as odd and hence the answer is 3.
- This works for all the numbers whose unit digit is 7.

**Question 3:** Find the remainder when  $52^{28}$  is divided by 6.

**Explanation :**

With any number divided by 6, the trick is the first remainder is the final remainder no matter the power of the number. So, 52 divided by 6 will give us a remainder of 4,  $52^2$  divided by 6 will give us a remainder of 4,  $52^3$  divided by 6 will give us a remainder of 4, any power would also give us the same remainder. So if  $49^{933}$  divided by 6, will always give a remainder of 1 and  $51^{45}$  divided by 6 will always give a remainder of 3.

## 1.9 ASSIGNMENT - NUMBER SYSTEM

1. What is X if  $8 - 1 * X = -4$  \* -1?
  - a. 2
  - b. -1
  - c. -2
  - d. 4
  - e. None of these
2. What is the least number which should be added to 1330 to make it a perfect square?
  - a. 56
  - b. 1
  - c. 41
  - d. 30
  - e. 39
3. Which of the following is the set of all positive integers?
  - a.  $\{1, 2, 3, \dots\}$
  - b.  $\{0, 1, 2, 3, 4, \dots\}$
  - c.  $\{2, 4, 6, 8, \dots\}$
  - d.  $\{2, 5, 7, 11, \dots\}$
  - e. None of these
4. Find the units place digit of  $2^{99}$ ?
  - a. 4
  - b. 2
  - c. 8
  - d. 6
  - e. 0
5. Find the value of x if  $82x \div 8 - 16 = 87$ 
  - a. 4
  - b. 0
  - c. 8
  - d. 2
  - e. None of the above
6. 4522 is not completely divisible by
  - a. 7
  - b. 17
  - c. 19
  - d. 21
  - e. None of these

7. Which of the following is true for the value of  $0.999\dots$ ?
- a. It is equal to 1                      b. It is less than 1                      c. It is equal to 0.9
- d. is equal to 0.99                      e. None of these
8. Which of the following numbers is the smallest number?
- a.  $1/12$                       b.  $1/6$                       c.  $1/4$
- d.  $1/3$                       e.  $1/8$
9. The sum of two numbers is 38 and their difference is 2. determine the numbers.
- a. 10,12                      b. 20,18                      c. 17,15
- d. 21, 19                      e. None of the mentioned
10. The number  $567xy$  is completely divisible by 30. the possible of  $x$  and  $y$  can be
- a. 2 and 0                      b. 1 and 0                      c. 0 and 0
- d. 0 and 1                      e. None of these
11. If  $1 = \frac{3}{4} \frac{y}{x}$  then
- a.  $x = \frac{3y}{4}$                       b.  $x = \frac{y}{3}$                       c.  $x = \frac{2}{3}y$
- d.  $x = \frac{2y}{3}$                       e. None of these
12. Find the smallest positive number which is exactly divisible by  $1/3, 1/2, 3/7, 4/11$ .
- a. 10                      b. 11                      c. 13
- d. 14                      e. None of these

13. What is the least number, which leaves a remainder 4 when divided by 6 and 5 when divided by 7?  
  
a. 82  
b. 42  
c. 45  
d. 47  
e. None of these
14. The HCF of two numbers is 11 and their LCM is 693. If one number is 77, find the other number.  
  
a. 7  
b. 9  
c. 63  
d. 99  
e. None of these
15. If the average marks of a class of 50 students in English is 25 and the average marks of the top 20 students is 40, then the average marks of the rest of the students is  
  
a. 20  
b. 30  
c. 15  
d. 10  
e. None of these
16. The average of 6 numbers is 30. If the average of the first four is 25 and that of last three is 35, the fourth number is  
  
a. 35  
b. 25  
c. 30  
d. 40  
e. None of these
17. The largest measuring cylinder that can accurately fill 3 tanks of capacity 98, 182 and 266 liters each, is of capacity:  
  
a. 14 liters  
b. 12 liters  
c. 28 liters  
d. 98 liters  
e. None of these



- 23.** The HCF and LCM of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is :
- a. 66                                      b. 88                                      c. 99
- d. 77                                      e. None of these
- 24.** Which least number must be subtracted from 1936 so that the remainder when divided by 9, 10, and 15 will leave in each case the same remainder?
- a. 75                                      b. 16                                      c. 48
- d. 39                                      e. None of these
- 25.** A number is to be multiplied by the fraction  $\frac{4}{5}$ . But Samir, by mistake, multiplied it by  $\frac{5}{4}$  and obtained the number 81 more than the correct one. What was the original number?
- a. 180                                      b. 120                                      c. 90
- d. 150                                      e. None of these

## Answer Keys

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans	Q No	Ans
1	d	6	d	11	a	16	b	21	c
2	b	7	b	12	e	17	a	22	d
3	a	8	a	13	e	18	c	23	e
4	c	9	b	14	d	19	a	24	c
5	a	10	c	15	c	20	b	25	a







## UNIT - 2

### PERCENTAGES, PROFIT AND LOSS

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#### Learning Objective

**After going through this unit, you will be able to understand:**

- Basic Calculation of Percentages
  - Fractions to Percentages
  - Percentage Increase or Decrease
  - Compound Growths
  - Approaches to solve questions on percentages
- 

#### 2.1 Theoretical Framework of Percentages

##### Percentages: Concepts & Tricks

- ❖ Percent stands for Per 100. Anything out of 100 is percent. Percentages is an important unit to measure performance of an individual, an organisation, nation as a whole.
- ❖ That decimal fraction which has 100 as its denominator is known as Percentage. The numerator of such a fraction is known as Rate Per Cent.
- ❖ 15 % and 15/100 mean one and the same quantity.
- ❖ Any number written in the form of a fraction with 100 as the denominator is a percentage.

**For example:**  $13 = 1300/100 = 1300\%$

$$3/5 = 60/100 = 60\%$$

$$62.5/100 = 62.5\%$$

Alternatively, X% of a number Y =  $(X \times Y)/100$

For example:  $16\frac{2}{3}\%$  of 300 =  $(50/3) \times (300/100) = 50$

❖ **A. Conversion from a Fraction to Percent and vice versa**

1. Fraction to Percent: Multiply the fraction by 100 to convert it into a percent.

E.g.:  $0.5 = 0.5 \times 100 = 50\%$

$$5/8 = 5/8 \times 100 = 62.5\%$$

2. Percent to Fraction: Reversing the earlier operation converts a percent to a fraction - i.e. divide the percent by 100.

e.g.  $60\% = 40/100 = 0.6$

$$65\% = 65/100 = 0.65 = 65:100$$

❖ **B. Percentage Increase or Decrease of a Quantity:**

Here, one point is to be noted, that the increase or the decrease is always on the original quantity. If the increase or decrease is given in absolute and the percentage increase or decrease is to be calculated, then the following formula is applied to do so.

$$\% \text{ increase / decrease} = 100 \times \text{Quantity increase or decrease} / \text{original quantity}$$

The point worth remembering is that the denominator is the ORIGINAL QUANTITY.

e.g. The salary of a Rakesh goes up from Rs 100 to Rs 135. What is the percentage increase in his salary?

$$\text{Increase} = 135 - 100 = \text{Rs. } 35. \therefore \% \text{ increase} = 35/100 \times 100\% = 35\%$$

Alternatively, if the salary of the man had been reduced from Rs. 145 to Rs 100, what is the percentage decrease in his salary?

$$\text{Decrease} = 145 - 100 = \text{Rs. } 45$$

$$\therefore \% \text{ decrease} = 45/145 \times 100\% = 31.03 \%$$

Note that for the same quantity of increase or decrease the % increase and % decrease have two different answers. The change in the denominator i.e. the original value changes in the above two situations and hence the difference.

❖ ***C. To Increase a Number by y %:***

If a number is increased by 10 %, then it becomes 1.1 times of itself.

If a number is increased by 30 %, then it becomes 1.3 times of itself.

❖ ***D. To Decrease a Number by y %:***

If a number is decreased by 10 %, then it becomes 0.90 times of itself.

If a number is decreased by 30 %, then it becomes 0.70 times of itself.

***E. Equivalent Percentages using Fractions:***

$\frac{1}{2}$	50%	$\frac{3}{4}$	75%	$\frac{2}{9}$	$22\frac{2}{9}\%$
$\frac{1}{3}$	$33\frac{1}{3}\%$	$\frac{4}{5}$	80%	$\frac{1}{15}$	$6\frac{1}{3}\%$
$\frac{1}{4}$	25%	$\frac{1}{8}$	$12\frac{1}{2}\%$	$\frac{1}{20}$	5%
$\frac{1}{5}$	20%	$\frac{1}{12}$	$8\frac{1}{3}\%$	$\frac{1}{25}$	4%
$\frac{1}{6}$	$16\frac{2}{3}\%$	$\frac{3}{8}$	$37\frac{1}{2}\%$	$\frac{1}{50}$	2%
$\frac{2}{5}$	40%	$\frac{5}{8}$	$62\frac{1}{2}\%$	$\frac{4}{3}$	$133\frac{1}{3}\%$
$\frac{3}{5}$	60%	$\frac{7}{8}$	$87\frac{1}{2}\%$	$\frac{5}{4}$	125%
$\frac{2}{3}$	$66\frac{2}{3}\%$	$\frac{1}{9}$	11.11%	$\frac{6}{5}$	120%

**❖ F. Income/Weight/Salary More**

If Pankaj's salary is R % more than Rohan, then Rohan's salary is less than that of Pankaj by  $100 \times R / (100 + R) \%$ .

## 2.2 SOLVED EXAMPLES - PERCENTAGES

**Question 1:** *What will be the fraction of 20%*

- a.  $\frac{1}{4}$
- b.  $\frac{1}{5}$
- c.  $\frac{1}{10}$
- d. None of above

**Explanation:**

It will  $20 \times \frac{1}{100} = \frac{1}{5}$

**Question 2:** *A fruit seller had some apples. He sells 40% apples and still has 420 apples. Originally, he had:*

- a. 588 apples
- b. 600 apple
- c. 672 apples
- d. 700 apples

**Explanation:**

- Suppose originally he had  $x$  apples.
- Then,  $(100 - 40)\%$  of  $x = 420$ .
- $\frac{60}{100} \times x = 420$
- $x = 420 \times \frac{100}{60} = 700$

**Question 3:** *In an election between two candidates, one got 55% of the total valid votes, 20% of the votes were invalid. If the total number of votes was 7500, the number of valid votes that the other candidate got, was*

- a. 2500
- b. 2700
- c. 2900
- d. 3100

**Explanation:**

- Total number of votes = 7500
- Given that 20% of Percentage votes were invalid
- Valid votes = 80%

- Total valid votes =  $7500 \times (80/100)$
- 1st candidate got 55% of the total valid votes.
- Hence the 2nd candidate should have got 45% of the total valid votes
- Valid votes that 2nd candidate got = total valid votes  $\times (45/100)$   
 $7500 \times (80/100) \times (45/100) = 2700$

**Question 4:** *Two students appeared at an examination. One of them secured 9 marks more than the other and his marks was 56% of the sum of their marks. The marks obtained by them are:*

- |           |           |
|-----------|-----------|
| a. 39, 30 | b. 41, 32 |
| c. 42, 33 | d. 43, 34 |

**Explanation:**

- Let their marks be  $(x + 9)$  and  $x$ .
- Then,  $x + 9 = 56/100(x + 9 + x)$
- $25(x + 9) = 14(2x + 9)$
- $3x = 99$
- $x = 33$
- So, their marks are 42 and 33.

**Question 5:** *The price of sugar having gone down by 10%, a consumer can buy 5 kg more sugar for Rs 270. The difference between the original and reduced price per kg is*

- |             |             |
|-------------|-------------|
| a. 62 paise | b. 60 paise |
| c. 75 paise | d. 53 paise |



**Explanation:**

- Reduced price  $\Rightarrow$  Rp/100y per kg
- $R = \text{Rs } 270$  ,  $p=10\%$  ,  $y=5\text{kg}$
- $\{(270 \times 10)/(100 \times 5)\} = \text{Rs. } 5.4$
- Original prices per kg =  $\text{Rp}/(100-p)y$  per kg
- $\{(270 \times 10)/(100 - 10)5\} = 2700/450 = \text{Rs. } 6$
- Difference will be  $= 6 - 5.4 = \text{Rs } 0.6 = 60 \text{ paise}$

**Question 6:** A shopkeeper bought 600 oranges and 400 bananas. He found 15% of oranges and 8% of bananas were rotten. Find the percentage of fruits in good condition.

- a. 23.4%
- b. 54.8%
- c. 64.5%
- d. 87.8%

**Explanation:**

- Total number of fruits shopkeeper bought  $= 600 + 400 = 1000$
- Number of rotten oranges  $= 15\%$  of 600  
 $= 15/100 \times 600$   
 $= 9000/100$   
 $= 90$
- Number of rotten bananas  $= 8\%$  of 400  
 $= 8/100 \times 400$   
 $= 3200/100$   
 $= 32$
- Therefore, total number of rotten fruits  $= 90 + 32 = 122$   
Therefore Number of fruits in good condition  $= 1000 - 122 = 878$   
Therefore Percentage of fruits in good condition  $= (878/1000 \times 100)\%$   
 $= (87800/1000)\% = 87.8\%$

**Question 7:** Three candidates contested an election and received 1136, 7636 and 11628 votes respectively. What percentage of the total votes did the winning candidate got

- a. 55%                      b. 56%
- c. 57%                      d. 58%

**Explanation:**

→ Total number of votes polled =  $(1136 + 7636 + 11628) = 20400$ . So, Required percentage =  $11628/20400 * 100 = 57\%$

**Question 8:** 2.09 can be expressed in terms of percentage as:

- a. 2.09%                      b. 20.9%
- c. 209%                        d. 0.209%

**Explanation:**

→ While calculation in terms of percentage we need to multiply by 100, so  $2.09 * 100 = 209$ .

**Question 9:** A reduction of 10% in the price of wheat enables a man to buy 50 g of wheat more for a rupee. How much wheat could originally be had for a rupee?

- a. 400g                      b. 500g  
c. 450g                      d. 350g

**Explanation:**

→ Original prices per kg =  $\frac{Rp}{(100-p)y}$  per kg

$$\{(1 \times 10)/(100 - 10)0.05\} = \text{Rs. } 20/9 \text{ /kg}$$

→ It means the man can buy 1 kg or 1000 gm in Rs 20/9 or he can buy

$$(1000 \times 9)/20 = 450\text{g in one rupee.}$$

**Question 10:** 75 g of sugar solution has 30% sugar in it. Then, the quantity of sugar that should be added to the solution to make the quantity of the sugar 70% in the solution, is

- a. 125 g
- b. 100 g
- c. 120 g
- d. 130 g

**Explanation:**

- Sugar in original solution =  $(75 \times 30) / 100 = 22.5$  g
- Let x g of sugar be mixed
- According to the question
- $\{(22.5 + p)/(75 + p)\} \times 100 = 70$
- $2250 + 100p = 75 \times 70 + 70p$
- $2250 + 100p = 5250 + 70p$
- $30p = 5250 - 2250 = 3000$
- $p = 3000/30 = 100$ g

**Question 11:** Half of 1 percent written as decimal is

- a. 5
- b. 0.5
- c. 0.05
- d. 0.005

**Explanation:**

- It will be  $1/2(1\%) = 1/2(1/100) = 1/200 = 0.005$

**Question 12:** A student has to obtain 33% of the total marks to pass. He got 125 marks and failed by 40 marks. The maximum marks are :

- a. 500
- b. 600
- c. 800
- d. 1000

**Explanation:**

- Given that the student got 125 marks and still he failed by 40 marks
- The minimum pass mark =  $125 + 40 = 165$
- Given that minimum pass mark = 33% of the total mark
- Total marks =  $165 / 0.33 = 500$

**Question 13:** *A man spends 35% of his income on food, 25% on children's education and 80% of the remaining on house rent. What percent of his income he is left with?*

- a. 6 %
- b. 8 %
- c. 10 %
- d. 12 %

**Explanation:**

- Let the total income be  $x$ . Then, income left =  $(100 - 80)\%$  of  $[100 - (35 + 25)]\%$  of  $x$
- $= 20\%$  of  $40\%$  of  $x = 20/100 \times 40/100 \times 100\%$  of  $x = 8\%$  of  $x$ .

**Question 14:** *If  $A = x\%$  of  $y$  and  $B = y\%$  of  $x$ , then which of the following is true?*

- a.  $A$  is smaller than  $B$ .
- b.  $A$  is greater than  $B$
- c. The relationship between  $A$  and  $B$  cannot be determined.
- d. If  $x$  is smaller than  $y$ , then  $A$  is greater than  $B$ .
- e. None of these

**Explanation:**

- $x\%$  of  $y = x/100 \times y = y/100 \times x = y\%$  of  $x$
- $A = B$ .

**Question 15:** If  $y$  exceeds  $x$  by 20%, then  $x$  is less than  $y$  by?

- a. 16%
- b.  $16\frac{1}{3}\%$
- c.  $16\frac{2}{3}\%$
- d.  $16\frac{3}{5}\%$

**Explanation:**

- $X=100$   $y=120$
- $120-----20$
- $100-----? \Rightarrow 16\frac{2}{3}\%$

**Question 16:** What will be the fraction of 4%

- a.  $\frac{1}{20}$
- b.  $\frac{1}{50}$
- c.  $\frac{1}{75}$
- d.  $\frac{1}{25}$

**Explanation:**

- $4 \times \frac{1}{100} = \frac{1}{25}$ .

**Question 17:** What is 15 percent of 34?

- a. 5.10
- b. 4.10
- c. 3.10
- d. 2.10

**Explanation:**

- It will be 15% of 34
- $(\frac{15}{100}) \times 34 = 5.10$

**Question 18:** In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is  $\frac{2}{3}$  of the number of students of 8 years of age which is 48. What is the total number of students in the school?

- a. 72
- b. 80
- c. 120
- d. 100

***Explanation:***

→ Let the number of students be  $x$ .

→ Then, Number of students above 8 years of age =  $(100 - 20) \%$  of  $x = 80\%$  of  $x$ .

→  $80\%$  of  $x = 48 + \frac{2}{3}$  of 48

→  $\frac{80}{100}x = 80$

$$x = 100$$

***Question 19:  $11 \frac{1}{9} \%$  of 900?***

a. 100

b. 20

c. 81

d. None of these

***Explanation:***

→  $11 \frac{1}{9} \% = \frac{1}{9}$

→  $\frac{1}{9} \times 900 = 100$

***Question 20: What percent is 70 of 280?***

a. 25%

b. 50%

c. 75%

c. None of these

***Explanation:***

→  $\frac{70}{280} = \frac{1}{4}$

→  $\frac{1}{4} \times 100 = 25 \%$

***Question 21: If A's height is 40% less than that of B, how much percent B's height is more than that of A?***

a. 66.66%

b. 76.66%

c. 96.66%

d. 86.66%

**Explanation:**

→ Excess of B's height over A's =  $[(40/(100 - 40))] \times 100\% = 66.66\%$

**Question 22:** *A number is decreased by 10% and then increased by 10%. The number so obtained is 10 less than the original number. What was the original number?*

- a. 1000
- b. 2000
- c. 3000
- d. 4000

**Explanation:**

→ Let the original number be x.

→ Final number obtained = 110% of (90% of x) =  $(110/100 \times 90/100 \times x) = (99/100)x$ .

→  $x - (99/100)x = 10$

→  $x = 1000$

**Question 22:** *The ratio 5:20 expressed as percent equals to*

- a. 50 %
- b. 125 %
- c. 25 %
- d. None of above

**Explanation:**

→ Actually, it means 5 is what percent of 20, which can be calculated as,  
 $(5/20) \times 100 = 5 \times 5 = 25$

## 2.3 ASSIGNMENT - PERCENTAGES

1. What percent is equivalent to 514?

- a. 500%
- b. 625%
- c. 600%
- d. 525%

2. What fraction is equivalent to 0.6%?

- a.  $\frac{3}{500}$
- b.  $\frac{3}{50}$
- c.  $\frac{3}{5}$
- d. None of these

3. What percent of 12 is 84?

- a. 70%
- b. 7000%
- c. 100%
- d. 700%

4.  $x\%$  of  $y + y\%$  of  $x = ?$

- a. 2% of  $xy$
- b. 100% of  $xy$
- c. 50% of  $xy$
- d.  $2xy$

5. What percent of 4 km is 8 m?

- a. 0.5%
- b. 2%
- c. 0.2%
- d. 20%

6. Find the number which is 60% less than 80.

- a. 20
- b. 32
- c. 40
- d. 39

7. 10% of 30% of 20% of 1000 = ?

- a.  $\frac{1}{6}$
- b. 6
- c. 6.5
- d. 0.6



8. The price of petrol has increased by 10%. By how much percent a person should reduce its consumption so as not to increase his expenditure?

- a. 90%                                      b. 100/11%
- c. 10%                                        d. 100/3%

9. If the salary of a person is first increased by 15% and then decreased by 12%, what is the percent change in his salary?

- a. 3% increase                      b. 1% decrease
- c. 1.2% increase                  d. 2% decrease

**10.** If the radius of a circle is increased by 4%, by what percent its area will increase?

- a. 4%    b. 8.16%
- c. 16%    d. 8%

**11.** The population of a town increases 5% annually. If its present population is 84000, what will it be in two years time?

- a. 92610                      b. 92600  
c. 90000                      d. 92200

**12.** Ram loses 20% of his pocket money. After spending 25% of the remainder, Rs.480 were left. What was his pocket money?

- a. Rs.900    b. Rs.500
- c. Rs.1000    d. Rs.800

**13.** To pass an examination, a candidate must get 60% marks. If he gets 120 marks and fails by 60 marks, find the total marks for the examination.

- a. 300  
b. 200  
c. 250  
d. 400

**14.** In a certain Mathematics examination, there were 2000 students, out of which 900 were boys and the rest were girls. If 32% of the boys and 38% of the girls passed, find the total percentage of failed students.

- a. 33.33%
- b. 45.67%
- c. 62.7%
- d. 64.7%

**15.** In 2005, Anil's salary was Rs.24000 and Sunil's salary was Rs.16000. In 2004, Sunil's salary was 25% of the sum of their salaries in 2005. What is the percent increase in Sunil's salary from 2004 to 2005?

- a. 16%
- b. 66%
- c. 60%
- d. 10%

**16.** At an election, the candidate who got 56% of votes cast won by 144 votes. Find total number of voters on the voting list if 80% people cast their vote and there were no invalid votes.

- a. 1200
- b. 1500
- c. 1800
- d. 2100

**17.** In a class, 25 % of the students were absent for an exam. 30% failed by 20 marks and 10% just passed because of grace marks of 5. Find the average score of the class if the remaining students scored an average of 60 marks and the pass marks are 33 (counting the final scores of the candidates)

- a. 36.93
- b. 32.05
- c. 28.2
- d. none of these

**18.**  $\frac{4}{5}$ th of the voters in Amethi promised to vote for Sonia Gandhi and the rest promised to vote for Sushma Swaraj. Of these voters, 10% of the voters who had promised to vote for Sonia Gandhi, did not vote on the Election Day while 20% of the voters who had promised to vote for Sushma Swaraj did not vote on the Election Day. What is the total number of votes polled if Sonia Gandhi got 216 votes?

- a. 250
- b. 264
- c. 300
- d. 295

**19.** Vimal's salary is 75% more than Kamal's salary. Vimal got a raise of 40% on his salary while Kamal got a raise of 25% on his salary. By what percent is Vimal's salary more than Kamal's?

- a. 96
- b. 90
- c. 86
- d. 50

**20.** A person started off a business with very little capital. In the first year he earned a profit of 50% and spent 50% of the total capital (initial capital + profit) for the sake of his business. The same procedure was followed in the 2nd and 3rd year. If at the end of 3 years he had left with Rs.16875. Find the amount remained at him at the end of 2nd year.

- a. 22,500
- b. 30,000
- c. 32,500
- d. 16875

**21.** The sum of the salaries of Suresh and Mahesh is 60,000. Being the best friends, they spend the same amount monthly and they save the remaining. If their savings are in the ratio 5:4, What is Suresh's salary?

- a. Rs.14,000
- b. Rs.28,000
- c. none of these
- d. data insufficient

**22.** In a museum, the entrance ticket costs Rs. 250. In vacations, the cost of the ticket is reduced thereby increasing the sale by 50%. But it was found that the collection is decreased by 17.5%. What is the deduction in the ticket price?

- a. Rs. 100
- b. Rs. 112.5
- c. Rs. 110
- d. Rs. 115.5

**23.** A cricket player won 80% of the matches he played. If he has played 35 matches, how many matches did he win?

- a. 28
- b. 25
- c. 20
- d. 32

**24.** A metal bar weighs 8.15 ounces. 93% of the bar is silver. How many ounces of silver are in the bar?

- a. 7.5
- b. 7.52

c. 7.57

d. 7.56

a. 45 %                      b. 45 5/11 %

c. 45 6/11 %                d. 46%

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans
1	d	6	b	11	a	16	b	21	d
2	a	7	b	12	d	17	c	22	b
3	d	8	b	13	a	18	b	23	a
4	a	9	c	14	d	19	a	24	c
5	c	10	b	15	c	20	a	25	b

## 2.4 PROFIT AND LOSS

### *Important Terminologies and Formulae:*

1. **Cost Price:** The price at which an article is purchased is called cost price.
2. **Selling Price:** The price at which an article is sold is called selling price (abbreviated as S.P.)
3. **Profit or Gain:** When an article is sold for more than what it costs we say there is profit or gain.

Thus, Profit = S.P. - C.P.

4. **Loss:** When an article is sold for less than what it costs we say there is loss. Thus, Loss = C.P. - S.P.
5. **Marked Price:** List price or the price printed on the article is known as marked price (abbreviated as M.P.)
6. **Discount:** Sometimes dealers allow some reductions on list price or marked price. This reduction is known as a discount.

### *Note:*

1. Profit percent or loss percent is always calculated over C.P. unless mentioned.
2. Discount is always calculated on Marked Price.

### *3.2 Useful Basic Relations*

1. **Profit (or Gain) %** :  $(\text{Profit}/\text{C.P.}) \times 100$
2. **Loss %** :  $(\text{Loss}/\text{C.P.}) \times 100$
3. **Successive Discounts** : If a tradesman marks his goods at a% above his cost price and allows purchasers a discount of b% for cash, then there is  $(a - b - (ab/100))$  % profit or loss, according to +ve or -ve sign respectively.
4. **If the SP of M articles = CP of N articles**, profit/loss% =  $[(N - M)/M] \times 100$ . In this if the answer is positive then it is profit and if it is negative then it is loss.

**5.** If the SP of two articles is the same and the loss percentage of one article is equal to the gain percentage of the other, then in such a case there is always net loss. And the net loss is given by the below formula:

$$\text{Loss} = [\text{common loss or gain \%}/10]^2$$

## 2.5 SOLVED EXAMPLES - PROFIT AND LOSS

**Question 1: Find profit or loss:  $C.P = Rs.870$  and  $S.P = Rs.770$**

- |           |           |
|-----------|-----------|
| a. RS.180 | b. Rs.150 |
| c. Rs.120 | d.Rs.100  |

**Explanation :**

- $C.P = RS.870$ ,  $S.P = Rs.770$  As,
- $C.P > S.P$  it means loss.
- $Loss = C.P - S.P = Rs.100$

**Question 2: If the cost price of a pencil box is Rs.67 and the selling price is Rs.70.Is there profit or loss?**

- |         |         |
|---------|---------|
| a. Rs.2 | b. Rs.3 |
| c. Rs.4 | d. Rs.5 |

**Explanation:**

- $C.P = Rs. 67$ ,  $S.P = Rs.70$
- As,  $S.P > C.P$  it means profit.
- $Profit = S.P - C.P = Rs.3$

**Question 3: A shopkeeper buys a TV at Rs.42000 and sells it at Rs.45000. What is his profit or loss?**

- |            |            |
|------------|------------|
| a. RS.2500 | b. Rs.3000 |
| c. Rs.4500 | d. RS.2000 |

**Explanation:**

- $C.P = Rs.42000$   $S.P = Rs.45000$
- As  $S.P > C.P$  So he makes profit
- $Profit = S.P - C.P = Rs.3000$



**Question 4:** Samant bought a microwave oven and paid 10 % less than the original price. He sold it with 30% profit on the price he had paid. What percentage of profit did Samant earn on the original price?

- a. 17%
- b. 20%
- c. 27%
- d. 32%

**Explanation:**

- Let the original price=Rs.100.
- Then, C.P = Rs.90.
- S.P =130% of Rs. 90=Rs. (130/100×90) = Rs.117.
- Required percentage=  $(117 - 100) \% = 17\%$ .

**Question 5:** If the cost price is 25% of selling price. Then what is the profit percent.

- A. 150%
- b. 200%
- C. 300%
- d. 350%

**Explanation:**

- Let the S.P = 100,
- then C.P. = 25
- Profit = 75
- Profit% =  $75/25 * 100 = 300\%$

**Question 6:** *The price of commodity X increases by 40 paise every year, while the price of commodity Y increases by 15 paise every year. If in 2001, the price of commodity X was Rs. 4.20 and that of Y was Rs. 6.30, in which year commodity X will cost 40 paise more than the commodity Y?*

- a. 2010
- b. 2011
- c. 2012
- d. 2013

**Explanation:**

- Suppose commodity X will cost 40 paise more than Y after z years.
- Then,  $(4.20 + 0.40z) - (6.30 + 0.15z) = 0.40$
- $0.25z = 0.40 + 2.10$
- $z = 2.50/0.25 = 250/25 = 10$ .
- X will cost 40 paise more than Y 10 years after 2001 i.e., 2011.

**Question 7:** *A vendor bought bananas at 6 for a rupee. How many for a rupee must he sell to gain 20%?*

- a. 3
- b. 4
- c. 5
- d. 6

**Explanation:**

- Let required number = x
- $20 = (6 \times 1/x \times 1 - 1.100)$
- $1/5 = 6/x - 1$
- $6/5 = 6/x$
- $x = 5$

**Question 8:** *The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, find the value of x?*

- a. 15
- b. 25

c. 18

d. 16

**Explanation:**

$$\rightarrow 25 = (20 - x)100/x$$

$$\rightarrow x = 4(20 - x)$$

$$\rightarrow 5x = 80$$

$$\rightarrow x = 16$$

**Question 9:** If a man reduces the selling price of a fan from Rs.400 to Rs.380, his loss increases by 2%. The cost price of the fan is

a. Rs. 480

b. Rs. 500

c. Rs. 600

d. Rs. 1000

**Explanation:**

$$\rightarrow \text{Let C.P be Rs. } x$$

$$\rightarrow \text{Then, } 2\% \text{ of } x = (400 - 380)$$

$$\rightarrow = 20$$

$$\rightarrow x/50 = 20$$

$$\rightarrow x = 1000.$$

**Question 10:** By selling 33 m of carpet, a man loses an amount equal to the selling price of 3 m of carpet. Find his gain or loss per cent.

a.  $8\frac{1}{3}\%$

b.  $\frac{2}{3}\%$

c.  $8\frac{1}{2}\%$

d.  $7\frac{3}{4}\%$

**Explanation:**

Let the SP of 1m carpet be Rs. x.

Therefore, by selling 33m cloth, Loss = 3x

S.P. of 33m cloth is 33x. and C.P. = S.P. + Loss. = 36x.

$$\text{Loss\%} = (3x/36x) * 100$$

i.e.  $8\frac{1}{3}\%$

**Question 11:** *A manufacturing company employs 10 machines, the aggregate output of which for a year is Rs 42000, and pays 10% of the profit to its shareholders. The manufacturing expenses for a year are Rs.1800 per machine and the establishment charges Rs 9000 per annum. What percent of the original profit will it pay to its shareholders, if one machine breaks down and are idle for one year?*

- a.  $7\frac{3}{10}\%$
- b. 3 %
- c.  $8\frac{2}{5}\%$
- d.  $9\frac{1}{3}\%$

**Explanation:**

- When all 10 machines are working, profit =  $42000 - [9000 - (1800 \times 10)] = \text{Rs.} 15000$ .
- If one breaks down, then the aggregate output =  $0.9 \times 42000 = \text{Rs.} 37800$ ,  
Manufacturing expenses = Rs 16200,  
Establishment charges = Rs.9000
- Therefore, Total expenses = Rs. 25200,  
Profit =  $\text{Rs.} 37800 - \text{Rs.} 25200 = \text{Rs.} 12600$
- The profit paid to shareholder is 10% of Rs 12600 = 1260  
Required percentage =  $1260 \times 100 / 15000 = 8\frac{2}{5}\%$

**Question 12:** *If a man reduces the selling price of a fan from Rs.400 to Rs.380, his loss increases by 2%. The cost price of the fan is:*

- a. Rs. 480
- b. Rs. 500
- c. Rs. 600
- d. Rs. 1000

**Explanation:**

- Let C.P be Rs. x
- Then,  $2\% \text{ of } x = (400 - 380) = 20$
- $x/50 = 20$
- $x = 1000$ .

**Question 13:** *A shopkeeper fixes the marked price of an item 35% above its cost price. The percentage of discount allowed to gain 8% is:*

- a. 20%
- b. 27%

c. 31%

d. 43%

***Explanation:***

→ Let C.P = Rs.100

→ Then, marked price = Rs.135,

→ S. P = Rs.108.

→ Discount % =  $(27/135 \times 100) \% = 20\%$

***Question 14: A kind of tape is selling at a profit of 12% of selling price, equal to 18 cents per yard. What is the selling price of the tape?***

a. \$ 1.50

b. \$ 1.00

c. \$ 1.20

d. none of these

***Explanation:***

→ Profit / % profit = selling price

→  $\$.18 / .12 = \$1.50$  selling price

**Question 15:** A merchant sold an article at 10% loss. If he had sold it Rs 450 more, 8% would have been gained on the cost price. Find the cost price?

- |            |            |
|------------|------------|
| a. Rs 1200 | b. Rs 1800 |
| c. Rs 1500 | d. Rs 2500 |

**Explanation:**

- Let the cost price be 100%. It is sold at 10% loss.
- So it is sold at 90% of the cost price.
- 90 % of the cost price + 450 = 108% of the Cost price
- 18% of the cost price = Rs 450
- Cost price of the book =  $450/18 \times 100 = \text{Rs } 2500$

**Question 16:** A trader mixes 26 kg of rice at Rs. 20 per kg with 30 kg of rice of other variety at Rs. 36 per kg and sells the mixture at Rs. 30 per kg. His profit percent is:

- |                       |        |
|-----------------------|--------|
| a. No profit, no loss | b. 5%  |
| c. 8%                 | d. 10% |

**Explanation:**

- C.P. of 56 kg rice = Rs.  $(26 \times 20 + 30 \times 36) = \text{Rs. } (520 + 1080) = \text{Rs. } 1600$ .
- S.P. of 56 kg rice = Rs.  $(56 \times 30) = \text{Rs. } 1680$
- Gain =  $n \left( \frac{80}{1600} \times 100 \right) \% = 5\%$

**Question 17:** Sam purchased 20 dozens of toys at the rate of Rs. 375 per dozen. He Sold each one of the rate of Rs. 33. What was his percentage profit?

- |        |        |
|--------|--------|
| a. 3.5 | b. 4.5 |
| c. 5.6 | d. 6.5 |

***Explanation:***

- Cost Price of 1 toy = Rs.  $(375/12) = \text{Rs. } 31.25$
- Selling Price of 1 toy = Rs. 33
- So, Gain = Rs.  $(33 - 31.25) = \text{Rs. } 1.75$
- Profit % =  $[(1.75/31.25) \times 100] \% = 28/5 \% = 5.6 \%$

***Question 18: The cost price of 20 articles is the same as the selling price of  $x$  articles. If the profit is 25%, find out the value of  $x$ ?***

- |       |       |
|-------|-------|
| a. 13 | b. 14 |
| c. 15 | d. 16 |

***Explanation:***

- If the SP of  $M$  articles = CP of  $N$  articles,  $\text{profit/loss}\% = [(N - M)/M] \times 100$ . In this if the answer is positive then it is profit and if it is negative then it is loss.
- So,  $[(20 - x)/x] \times 100 = 25$
- $x = 80 - 4x$ . Hence,  $x = 16$

a. 30%                      b. 15%

c.  $3\frac{1}{2}\%$                   d. 31%



**9.** A cloth merchant sells his goods at the cost price, but uses a faulty meter rod. If the rod measures 95 cm only, find his gain per cent.

- a. 100/29%
- b. 100/17%
- c. 100/21%
- d. 100/19%

**10.** What is the single discount equivalent to two successive discounts of 10% and 20%?

- a. 30%
- b. 15%
- c. 28%
- d. 25%

**11.** There are two shop-keepers who sell cupboards at the same list price. The first gives two successive discounts of 30% and 16% and the second gives 20% and 26%. Which discount series is profitable for the shop-keeper?

- a. First
- b. second
- c. both are same
- d. cannot be determined

**12.** Mr. Sharma sells a table at 10% below cost price. Had he received Rs.1494 more, he would have made a profit of 12.5%. Find the cost price of the table.

- a. Rs.6600
- b. Rs.6000
- c. Rs.6660
- d. Rs.6640

**13.** What should be the selling price of 250 chairs at Rs.50 per chair if the shop keeper declares the three successive discounts, 20%, 15%, 5%?

- a. Rs.7500
- b. Rs.8075
- c. Rs.8500
- d. Rs.9000

**14.** By selling 33 meters of cloth, a shop-keeper gains the price of 11 meters of cloth. What is his gain per cent?

- a. 33.33%
- b. 33%
- c. 50%
- d. 100%

**15.** If selling price is doubled, profit triples. What is the profit percent?

- a. 66.66%
- b. 33.33%
- c. 50%
- d. 100%

**16.** A person brought 20lts of milk at the rate of 8per liter. He got it churned after spending Rs.10 and 5kg of cream and 20lts of toned milk are obtained. If he sold the cream at Rs30 per kg and toned milk at Rs.4 per liter. His profit in the transaction is

- a. 25 %
- b. 35.2 %
- c. 35.7 %
- d. 42.2 %

**17.** Mr. Smith sold his car to his neighbor, Mr. John, and received a profit of 35%. But after a few months, Mr. John decided to purchase a new one and hence he sold the car back to Mr. Smith with 20% profit. Find percent loss or gain obtained by Mr. Smith in the transaction.

- a. 15% loss
- b. 55% gain
- c. 27% loss
- d. none of these

**18.** A whole-sale dealer made an advertisement that he takes only 1% profit on the cost price of the cloth. But after a huge sale in Diwali, he actually obtained a profit of 10% on the overall transaction by using a faulty meter rod. What was the length of the meter rod used by him?

- a. 91 cm
- b. 91.50 cm
- c. 91.80 cm
- d. 91.81 cm

**19.** A person purchased a carpet on which he got three successive discounts of 20%. If, after discount, he has paid a total amount of Rs. 1024 to the shop-keeper, what was the actual cost of the carpet?

- a. Rs. 1500
- b. Rs. 1800
- c. Rs. 2000
- d. Rs. 2200

**20.** Veena has a tea powder business. She purchases tea powder from a local wholesaler of a particular cost per kg. The price of the tea powder of her stores is Rs.3 per kg. Her faulty spring balance reads 0.9 kg for a KG. Also in the festival season, she gives a 10% discount on the tea powder. She found that she made neither a profit nor a loss in the festival season. At what price did Veena purchase the tea powder from the wholesaler?

- a. 3
- b. 2.5
- c. 2.43
- d. 2.7

**21.** Two sets of cassette player were sold for Rs. 4000. One was sold for 15% loss and the other for 15% gain. What was the net gain or net loss in percent?

- a. Loss 2.25%
- b. Gain 2.25%
- c. Gain 5%
- d. Loss 5%

**22.** Two washing machines were sold for Rs. 12000. One was sold at 20% loss and the other at 20% profit. What was the net gain or net loss in percent?

- a. Gain 4%
- b. Loss 4%
- c. Loss 6.25%
- d. Gain 6.25%

**23.** If two dozen mangoes were sold at the cost price of three dozen mangoes. Find the percentage loss or gain.

- a. 50% Gain
- b. 50% Loss

c. 40% Gain

d. 40% Loss

**24.** If strawberries weighing one quintal were sold at the cost price of strawberries weighing three-fourth of the quintal find the net gain or loss in percent.

a. 25% Gain

b. 25% Loss

c. 30% Loss

d. 30% Gain

**25.** If three dozen pencils were purchased for Rs. 72 and were sold at the price of Rs. 72 for two dozen. Find the percentage profit or loss.

a. 40% Profit

b. 50% Loss

c. 50% Profit

d. 40% Loss

### 2.6.1 Answer Keys - Profit and Loss

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans
1	c	6	b	11	b	16	b	21	a
2	a	7	b	12	d	17	d	22	b
3	c	8	c	13	b	18	d	23	a
4	c	9	d	14	c	19	c	24	b
5	a	10	c	15	d	20	c	25	c



## **UNIT - 3**

### **SIMPLE INTEREST AND COMPOUND INTEREST**

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#### **Learning Objective**

**After going through this unit, you will be able to:**

- Basic Calculation of Simple and Compound Interest
  - Understand of Important concepts and formulas
  - Understand difference between SI and CI
  - Understand approaches of solving questions on interest
- 

#### **3.1 Theoretical Framework of Simple Interest**

##### **What is Simple Interest?**

Simple Interest (S.I) is the method of calculating the interest amount for some principal amount of money. Have you ever borrowed money from your siblings when your pocket money got exhausted? Or lent him maybe? What happens when you borrow money? You use that money for the purpose you had borrowed it in the first place. After that, you return the money whenever you get the next month's pocket money from your parents. This is how borrowing and lending work at home. But in the real world, money is not free to borrow. You often have to borrow money from banks in the form of a loan. During payback, apart from the loan amount, you pay some more money that depends on the loan amount as well as the time for which you borrow. This is called simple interest. This term finds extensive usage in banking.

## Simple Interest Formula

The Formula for simple interest helps you to find the interest amount if the principal amount, rate of interest and time periods are given.

Simple interest formula is given as

$$SI = (P \times R \times T) / 100$$

Where SI = simple interest

P = principal

R = interest rate (in percentage)

T = time duration (in years)

In order to calculate the total amount, the following formula is used:

$$\text{Amount (A)} = \text{Principal (P)} + \text{Interest (I)}$$

Where,

**Amount (A)** is the total money paid back at the end of the time period for which it was borrowed.

## Difference Between Simple Interest and Compound Interest

There is another type of interest called compound interest. The major difference between simple and compound interest is that simple interest is based on the principal amount of a deposit or a loan whereas the compound interest is based on the principal amount and interest that accumulates in every period of time. Let's see one simple example to understand the concept of simple interest.

### 3.2 SOLVED EXAMPLES - SIMPLE INTEREST

**Question 1:** *At what rate percent per annum using simple interest will a sum of money double in 8 years*

- a. 12.5%
- b. 13.5%
- c. 11.5%
- d. 14.5%

**Explanation:**

- Let principal = P, Then, S.I.= P and Time = 8 years
- We know that  $S.I. = \frac{PTR}{100}$
- Rate =  $[(100 \times P) / (P \times 8)] \% = 12.5\%$  per annum

**Question 2:** *A sum of money at simple interest amounts to Rs. 815 in 3 years and to Rs. 854 in 4 years. The sum is:*

- a. Rs. 650
- b. Rs. 690
- c. Rs. 698
- d. Rs. 700

**Explanation:**

- S.I. for 1 year = Rs.  $(854 - 815) = \text{Rs. } 39$ .
- S.I. for 3 years = Rs.  $(39 \times 3) = \text{Rs. } 117$ .
- Principal = Rs.  $(815 - 117) = \text{Rs. } 698$ .

**Question 3:** *What is the rate of simple interest?*

- I. The total interest earned was Rs. 4000.
- II. The sum was invested for 4 years.

*a. Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.*

b. Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question.

c. Statement I or in Statement II alone are sufficient to answer the question.

d. Statements I and II together are not sufficient to answer the question.

**Explanation:**

- We know that,  $R = (100 * S.I.) / (P * T)$ .
- Now, I give, S.I. = Rs. 4000.
- II gives, T = 4 years.
- But, P is unknown. So, we cannot find R.
- So, given data is insufficient to get R.

**Question 3:** A sum of Rs. 25000 becomes Rs. 27250 at the end of 3 years when calculated at simple interest. Find the rate of interest.

- a. 2%
- b. 3%
- c. 4%
- d. 5%

**Explanation:**

- Simple interest =  $27250 - 25000 = 2250$
- Time = 3 years.
- $SI = PTR / 100$  and so  $R = SI * 100 / PT$
- $R = 2250 * 100 / 25000 * 3$ , so,  $R = 3\%$ .

**Question 4:** Ariel takes a loan of \$8,000 to buy a used truck at the rate of 9 % simple Interest. Calculate the annual interest to be paid for the loan amount.

- a. 720
- b. 890
- c. 688
- d. 700

**Explanation:**

- From the details given in the problem Principle =  $P = \$8,000$  and  $R = 9\%$  or 0.09 expressed as a decimal.



- As the annual Interest is to be calculated, the time period  $T = 1$ .
- Plugging these values in the simple Interest formula,
- $I = P \times T \times R$
- $8,000 \times 1 \times 0.09$
- 720.00

***Question 5: In what time will the simple interest on Rs 400 at 10% per annum be the same as the simple interest on Rs 1000 for 4 years at 4 % per annum?***

- a. 2 yrs
- b. 3 yrs
- c. 4 yrs
- d. 6 yrs

***Explanation:***

- Here,  $P = \text{Rs } 1000$ ,  $T = 4 \text{ yrs}$  and  $R = 4 \%$
- Since, Simple Interest on Rs 1000 =  $(1000 \times 4 \times 4)/100 = \text{Rs } 160$
- Now, simple interest = Rs 160
- $P = \text{Rs } 400$
- $R = 10 \%$  then,  $T = (100 \times SI)/P \times R$
- $(100 \times 160) / (400 \times 10)$
- 4 yrs

**Question 6:** A financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes.

- a. 10.25%
- b. 10%
- c. 9.25%
- d. 9%

**Explanation:**

- Let the sum is 100.
- As financier includes interest every six months., then we will calculate SI for 6 months, then again for six months as below:
- SI for first Six Months =  $(100 \times 10 \times 1) / (100 \times 2) = \text{Rs. } 5$
- Important: now sum will become  $100 + 5 = 105$
- SI for last Six Months =  $(105 \times 10 \times 1) / (100 \times 2) = \text{Rs. } 5.25$
- So amount at the end of year will be  $(100 + 5 + 5.25) = 110.25$   
 $\text{Effective rate} = 110.25 - 100 = 10.25$

**Question 7:** A man borrowed some amount, after 3 years he paid Rs. 10,400/- with 10% interest, then how much Amount he borrowed?

- a. Rs.7000/-
- b. Rs.8000/-
- c. Rs.9000/-
- d. Rs.1000/-

**Explanation:**

- Rate of interest  $R = 10\%$ , Time  $T = 3$  years, Accumulated Amount  $A = \text{Rs. } 10400/-$
- Let the Principal be Rs.100. So, SI for 3 years will be:  $\text{Rs. } 30 \dots [(100 \times 10 \times 3) / 100]$
- So, if P is 100, Amount is 130
- If Amount is 10400, P will be  $(10,400 \times 100 / 130)$
- Rs. 8000

**Question 8:** A person took some amount with some interest for 2 years, but increase the interest for 1%, he paid Rs.120/- extra, then how much amount he took?

- a. Rs.325                                      b. Rs.545  
c. Rs.560                                      d. Rs.550

***Explanation:***

- Let each instalment be Rs.  $x$ .
- 1st year =  $[x + (x * 12 * 2)/100]$
- 2nd year =  $[x + (x * 12 * 1)/100]$
- 3rd year =  $x$
- Then,  $[x + (x * 12 * 2)/100] + [x + (x * 12 * 1)/100] + x = 1092$
- $3x + (24x/100) + (12x/100) = 1092$
- $336x = 109200$
- Therefore,  $x = 325$
- Each instalment = Rs. 325

***Question 12: A sum fetched a total simple interest of 4016.25 at the rate of 9 % p.a. in 5 years.***

***What is the sum?***

- a. 4462.50
- b. 8032.50
- c. 8925
- d. None of these

***Explanation:***

- Principal =  $(100 \times 4016.25) / (9 \times 5) = 401625/45 = 8925$ .

***Question 13: Find the rate at Simple interest, at which a sum becomes four times of itself in 15 years***

- a. 10%
- b. 20%
- c. 30%
- d. 40%

***Explanation:***

- Let sum be  $x$  and rate be  $r\%$  then,  $(x * r * 15)/100 = 3x$  [important to note here is that simple interest will be  $3x$  not  $4x$ , because  $3x + x = 4x$ ]
- $r = 20\%$

**Question 14:** *In 10 years, A wants simple interest to be half of the amount. So, what should be the rate percent per annum?*

- a. 5.66%
- b. 5%
- c. 4%
- d. 6%

**Explanation:**

- Let sum = x. Then, S.I. =  $x/2$
- $SI = \frac{PRN}{100}$
- $x/2 = x * R * 10 / 100$
- Rate =  $(100 * 2x/x * 5 * 10) \% = 4\%$

**Question 15:** *A loan taken by a man, for every 5 years it will be doubled, then how much time it will be taken*

- a. 12 years
- b. 15 years
- c. 16 years
- d. 13 years

**Explanation:**

- (R)1% -----> 5 years
- $3/1\% \times 5 = 15$  years
- = 15 years

**Question 16:** A loan taken by a man, for every 8 years it will be 3 times, then what is the rate of the interest?

- a. 15%
- b. 20%
- c. 25%
- d. 28%

**Explanation:**

→  $P = 1, I = 2, T = 8 \text{ years } R = ?$

→  $I = PTR/100$

→  $2 = 1 \times 8 \times R/100$

→  $R = 25\%$

**Question 17:** A sum of money at simple interest amounts to Rs. 2240 in 2 years and to Rs. 2600 in 5 years. What is the principal amount?

- a. 1000
- b. 1500
- c. 2000
- d. 2500

**Explanation:**

→ SI for 3 years =  $2600 - 2240 = 360$

→ SI for 2 year  $360/3 \times 2 = 240$

→ Principal =  $2240 - 240 = 2000$

**Question 18:** Simple interest on a certain sum is  $16/25$  of the sum. Find the rate percent and time, if both are numerically equal?

- a. Rate = 7% and Time = 7 years
- b. Rate = 8% and Time = 8 years
- c. Rate = 6% and Time = 6 years
- d. Rate = 5% and Time = 5 years

**Explanation:**

→ Let sum = X. Then S.I =  $16x/25$

→ Let rate = R% and Time = R years

→ Therefore,  $(x \times R \times R)/100 = 16x/25 \Rightarrow R = 40/5 = 8$

→ Therefore, Rate = 8% and Time = 8 years

**Question 19:** In how many years Rs 150 will produce the same interest at 8% as Rs. 800 produce in 3 years at 9/2%

- a. 8
- b. 9
- c. 10
- d. 11

**Explanation:**

→ Firstly, we need to calculate the SI with principle 800, Time 3 years and Rate 9/2%, it will be Rs. 108.

Then we can get the Time as  $\text{Time} = (100 \times 108) / (150 \times 8) = 9$

**Question 20:** How much time will it take for an amount of Rs. 450 to yield Rs. 81 as interest at 4.5% per annum of simple interest?

- a. 3.5 years
- b. 4 years
- c. 4.5 years
- d. 5 years

**Explanation:**

→  $\text{Time} = (100 \times 81 / 450 \times 4.5) = 4 \text{ Years.}$

**Question 21:** Ramya gives 50 times the rent per annum to purchase a plot from L.I.C. Find the rate of interest from the amount paid by him.

- a. 1%
- b. 2%
- c. 3%
- d. 1.5%

**Explanation :**

→ Let the annual rent be Rs. x.  $I = x$ ;  $P = 50x$ ,  $n = 1$

→ The rate of interest  $= 100I/Pn = 100x/(50x \times 1)\% = 2\%$

**Question 22:** *The sum required to earn a monthly interest of Rs 400 at 10 % per annum at simple interest is:*

- |             |             |
|-------------|-------------|
| a. Rs 2000  | b. Rs 12000 |
| c. Rs 24000 | d. Rs 48000 |

**Explanation:**

→ Total interest needed in a year = Rs 400 × 12 = Rs 4800

→  $SI = (P \times R \times T)/100$

→  $4800 = (P \times 10 \times 1)/100$

→  $P = \text{Rs.} 48000$

**Question 23:** *The simple interest on Rs. 1820 from March 9, 2005, to May 21, 2005, at 7.5% rate will be:*

- |              |              |
|--------------|--------------|
| a. Rs. 28.50 | b. Rs. 27.30 |
| c. Rs. 23.80 | d. Rs. 30    |

**Explanation:**

Time = (22 + 30 + 21) days = 73 days =  $\frac{1}{5}$  years

S.I =  $\text{Rs.} 1820 \times \frac{15}{2} \times \frac{1}{5} \times \frac{1}{100} = \text{Rs.} 27.50$



### 3.3 ASSIGNMENT - SIMPLE INTEREST

1. Find the simple interest on Rs.5200 for two years at the rate of interest at 6% per annum.

- a. Rs.624
- b. Rs. 620
- c. Rs. 635
- d. Rs. 615

2. A man earns Rs.450 as an interest in 2 years on a certain sum at the rate of 12% per annum on SI. Find the money invested.

- a. Rs.1775
- b. Rs. 1567
- c. Rs. 1875
- d. Rs. 1234

3. Find the rate of interest p.a. with SI, which the sum of Rs.5000 will amount to Rs.6000 in 4 years.

- a. 4.5%.
- b. 5%
- c. 5.5%
- d. 8%

4. Find the time in which there will be an interest of Rs.240 on Rs.1200 at 5% per year using SI.

- a. 4 years.
- b. 5 years
- c. 6 years
- d. 3 years

5. A person borrows Rs.3000 from his friend at 15% per year for 3 years. What is the interest and the money to be returned respectively by the person to his friend use SI?

- a. Rs.2350 and Rs.5350
- b. Rs.1360 and Rs.4450
- c. Rs.1480 and Rs.4860
- d. Rs.1350 and Rs.4350

6. What principal will amount to Rs.570 at 4% per annum in 5 years using SI?

- a. Rs. 485
- b. Rs. 500
- c. Rs. 475
- d. Rs. 510

7. A certain sum of money trebles itself in 5 years of simple interest. Find the rate per cent per annum.

- a. 43%.
- b. 40%
- c. 42%
- d. 45%

8. In what time a sum of money will double itself at a rate of simple interest of 8% per annum?

- a. 10 years
- b. 18 years
- c. 12 years
- d. 12.5 years

**9.** A sum of money lent on simple interest, doubles itself in 15 years. In how many years would it treble itself?

- a. 30 years  
c. 24 years  
b. 26 years  
d. 25.5 years

**10.** If a certain sum of money at simple interest amounts to Rs.5184 in two years and to Rs.5832 in 3 years, what is the rate of interest?

- a. 50/6 %  
c. 50/7 %  
b. 50/3 %  
d. 50/11 %

### ***3.3.1 Answer Keys - Simple Interest***

<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>
<b>1</b>	a	<b>6</b>	c
<b>2</b>	c	<b>7</b>	b
<b>3</b>	b	<b>8</b>	d
<b>4</b>	a	<b>9</b>	a
<b>5</b>	d	<b>10</b>	b

### 3.4 COMPOUND INTEREST - CONCEPT AND FORMULA

Compound interest is the interest calculated on the principal and the interest accumulated over the previous period. It is different from the simple interest where interest is not added to the principal while calculating the interest during the next period. Compound interest finds its usage in most of the transactions in the banking and finance sectors and also in other areas as well. Some of its applications are:

1. **Increase or decrease in population.**
2. **The growth of bacteria.**
3. **Rise or depreciation in the value of an item.**

#### Compound Interest Maths

To understand the compound interest, we need to do its Mathematical calculation. To calculate compound interest, we need to know the amount and principal. It is difference between amount and principal.

#### Compound Interest Formula

The compound interest formula is given below:

$$\text{Compound Interest} = \text{Amount} - \text{Principal}$$

Where the **amount** is given by:

Where,

A = amount

P = principal

R = rate of interest

t = number of times interest is compounded per year

It is to be noted that the above formula is the general formula for the number of times the principal is compounded in a year. If the amount is compounded annually, the amount is given as-

$$A = P (1 + R/100)^t$$

### 3.5 SOLVED EXAMPLES - COMPOUND INTEREST

**Question 1:** Find compound interest on Rs. 8000 at 15% per annum for 2 years and 4 months, compounded annually.

- a. 2109
- b. 3109
- c. 4109
- d. 6109

**Explanation:**

- Time = 2 years 4 months =  $2\frac{4}{12}$  years =  $2\frac{1}{3}$  years.
- Amount = Rs'.  $[8000 \times (1 + \frac{15}{100})^2 \times (1 + (\frac{1}{3} \times \frac{15}{100}))]$
- Rs.  $[8000 \times (\frac{23}{20}) \times (\frac{23}{20}) \times (\frac{21}{20})]$
- Rs. 11109.
- C.I. = Rs.  $(11109 - 8000) = \text{Rs. } 3109$ .

**Question 2:** Find the compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.

- a. Rs. 610
- b. Rs. 612
- c. Rs. 614
- d. Rs. 616

**Explanation:**

- Amount =  $[7500 \times (1 + \frac{4}{100})^2] = (7500 \times \frac{26}{25} \times \frac{26}{25}) = 8112$

**Question 3:** The simple interest on a sum of money for 3 years at  $6\frac{2}{3}\%$  per annum is \$ 6750. What will be the compound interest on the same sum at the same rate for the same period, compounded annually?

- a. 8125
- b. 6655
- c. 4496
- d. 7210

**Explanation:**

- Given, SI = \$ 6750, R =  $20\frac{2}{3}\%$  p.a. and T = 3 years.  
Sum =  $100 \times \text{SI} / \text{R} \times \text{T}$
- \$  $(100 \times 6750 \times \frac{3}{20} \times \frac{1}{3}) = \$ 33750$ .

- Now,  $P = \$ 33750$ ,  $R = 20\% \text{ p.a.}$  and  $T = 3 \text{ years.}$
- Therefore, amount after 3 years
- $= \$ \{33750 \times (1 + (20/3 \times 100))\}^3$  [using  $A = P (1 + R/100)$ ]
- $= \$ (33750 \times 16/15 \times 16/15 \times 16/15) = \$ 40960$
- Thus, amount = \$ 40960
- Hence, compound interest = \$ (40960 - 33750) = \$ 7210

**Question 4:** *If a sum of Rs.8000 was lent for an interest of 20% per annum then the sum of the amount will be Rs.13824 in:*

- |            |           |
|------------|-----------|
| a. 2 years | b. 1year  |
| c. 3years  | d. 4years |

**Explanation:**

- These types of examples can be solved by using line approach.
- 8000 ----+20% (1600) ----> 9600.....+20% (1920)....> 11520.....+20% (2304).....> 13824.
- So, it takes 3 years.

**Question 5:** *In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded annually?*

- |           |           |
|-----------|-----------|
| a. 1 year | b. 4 year |
| c. 3 year | d. 6 year |

**Explanation:**

- Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a. Let the time be n years.
- Then,  $[1000 (1 + (10/100))^n] = 1331$  or  $(11/10)^n = (1331/1000) = (11/10)^3$   
 $n = 3 \text{ years.}$

**Question 6: What is the difference between the compound interests on Rs. 5000 for 1.5 years at 4% per annum compounded yearly and half-yearly?**

- |             |             |
|-------------|-------------|
| a. Rs. 2.04 | b. Rs. 4.80 |
| c. Rs. 3.06 | d. Rs. 8.30 |

**Explanation:**

- CI on 5000 at 4% p.a. compounded yearly will be:
- $5000 \dots + 4\% (200) \dots > 5200 \dots + 2\% (104) \dots > 5304$
- CI on 5000 at 4% p.a. compounded half-yearly will be: (Rate would be 2% for half year now)
- $5000 \dots + 2\% (100) \dots > 5100 \dots + 2\% (102) \dots > 5202 \dots + 2\% (104.4) \dots > 5306.4$
- Difference in CI when compounded annually and semiannually be:
- $[(200+104) - (100+102+104.4)] = 2.04$

**Question 7: What would be the compound interest on Rs.7700/- at 15 ¼% per annum for 2 years compounded annually**

- |                 |                 |
|-----------------|-----------------|
| a. Rs.2725.75/- | b. Rs.2527.57/- |
| c. Rs.2227.57/- | d. Rs.2520.57/- |

**Explanation:**

- Principal = Rs. 7700/-, Time = 2 years, Rate = 15 ¼%
- Amount =  $P(1+R/100)^n$
- $= 7700 \times (1 + 61/(4 \times 100))^2$
- $= 7700 \times [(1 + 61/400)^2]$
- $= 7700 \times [(461/400)^2]$
- $= 7700 \times 461/400 \times 461/400$
- $= 7700 \times 1.1525 \times 1.1525$
- $= 7700 \times 1.32825625$
- $= 10227.573125$
- c.I =  $10227.573125 - 7700 = 2527.57/-$

**Question 8:** *There is 80% increase in an amount in 8 years at simple interest. What will be the compound interest of 80% increase in an amount in 8 years at simple interest? What will be the compound interest of Rs. 14,000 after 3 years at the same rate?*

- a. Rs.3794
- b. Rs.3714
- c. Rs.4612
- d. Rs.4634

**Explanation:**

- Let  $P = \text{Rs.}100$
- Simple Interest = Rs. 80 (80% increase is due to the simple interest)
- Rate of interest  $= (100 \times \text{SI}) / PT = (100 \times 80) / (100 \times 8) = 10\%$  per annum
- Now let's find out the compound interest of Rs. 14,000 after 3 years at 10%
- $P = \text{Rs.}14000$ ,  $T = 3$  years,  $R = 10\%$
- Amount after 3 years  $= P(1 + R/100)^T$   
 $= 14000(1 + 10/100)^3$   
 $= 14000(110/100)^3$   
 $= 14000(11/10)^3$   
 $= 14 \times 11^3 = 18634.$
- Compound Interest = Rs.18634 - Rs.14000 = Rs.4634

**Question 9:** *The compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is:*

- a. 2
- b.  $2 \frac{1}{2}$
- c. 3
- d. 4

**Explanation:**

- Amount = Rs. (30000 + 4347) = Rs. 34347
- Let the time be  $n$  years.
- Then,  $30000(1 + 7n/100) = 34347$
- $(107/100)^n = 34347/30000 = 11449/10000 = (107/100)^2$
- $n = 2$  years

**Question 10:** You have an investment proposition in front of you where you would invest 100 dollars in a project and after 2 years you can sell the investment. You would earn 5% paid annually OR you can receive interest quarterly. Which one should you go for?

- a. annually as you receive more in one go
- b. makes no difference as the investment is the same
- c. quarterly as you receive more
- d. depends on future interest rate variations

**Explanation:**

→ It is better to receive the interest quarterly as then you are receiving money more frequently and can reinvest that right away after quarter 1 instead of waiting the full year and then reinvest.

**Question 11:** A sum of money amounts to Rs.6690 after 3 years and to Rs. 10,035 after 6 years on compound interest. Find the sum.

- a. 4360
- b. 4460
- c. 4560
- d. 4660

**Explanation:**

→ Let the sum be Rs. P. Then  $P(1+R/100)^3=6690\dots(i)$

→ And  $P(1+R/100)^6=10035\dots(ii)$

→ On dividing, we get  $(1+R/100)^3=10025/6690=3/2$ .

→ Substituting this value in (i), we get:

→  $P*(3/2)=6690$  or  $P=(6690*2/3)=4460$

Hence, the sum is rs.4460.

**Alternative Method:**

→ Let the sum be Rs. P, Then  $10035/6690 = 6690/x$

→ Solving this,  $x = 4460$



a. Rs.420.20                      b. Rs.319.06

c. Rs.306.04                      d. Rs.294.75

$$A = 5000(51/50)^3 = 5306.04 = 5000 + 306.04$$

a. Rs. 99                                  b. Rs. 94.50

c. Rs. 95.60                              d. Rs. 108

→ Sum = Rs. 100 x 90/20 = Rs. 450

→ C.I. = Rs.450 x (1+ 10/100) ^2- 450 = Rs. 94.50

a. Nil  
b. Rs. 6.60  
c. Rs. 4.40  
d. Rs. 1.50

→ S.I. = Rs.  $(600 \times 5 \times 2)/100$  = Rs.60  
→ C.I.= Rs.  $[600 \times (1 + 5/100)^2 - 600]$  = Rs. 61.50  
→ Required Difference = Rs.  $(61.50 - 60)$  = Rs.1.50

a. 72 cm                      b. 74 cm

c. 75 cm

d. 81 cm

**Explanation:**

$$\rightarrow \text{Increase\%} = (1/8) \times 100\% = 12.5\%$$

$$\begin{aligned}\rightarrow \text{Height after 2 years} &= 64 \times \{1 + 25/(2 \times 100)\}^2 \\ &= 64 \times 9/8 \times 9/8 \\ &= 81 \text{ cm}\end{aligned}$$

**Question 16:** *The difference between the compound interest compounded annually and the simple interest on a certain sum for 2 years at 6% per annum is \$ 18. Find the sum.*

a. 5000

b. 6000

c. 7000

d. 8000

**Explanation:**

$$\rightarrow \text{Difference of CI and SI for 2 years} = PR^2/100^2$$

$$\rightarrow 18 = P \times 36/100$$

$$\rightarrow P = 18 \times 10000/36$$

$$\rightarrow P = 5000$$

**Question 17:** *In this question, the formula is when the interest is compounded annually to solve this problem of compound interest. 4. At what rate per cent per annum will Ron lends a sum of \$2000 to Ben. Ben returned after 2 years \$2205, compounded annually?*

a. 7%

b. 5%

c. 6%

d. 4%

**Explanation:**

$$\rightarrow \text{Let the required rate be } R\% \text{ per annum}$$

$$\rightarrow \text{Here, } A = \$2205, P = \$2000 \text{ and } n = 2 \text{ years}$$

$$\rightarrow \text{Using the formula, } A = P (1 + R/100)^n,$$

$$\rightarrow 2205 = 2000 \times (1 + R/100)^2$$

$$\rightarrow (1 + R/100)^2 = 2205/2000 = 441/400 = (21/20)^2$$

$$\rightarrow (1 + R/100) = 21/20$$

$$\rightarrow R/100 = (21/20 - 1) = 1/20$$

$$\rightarrow R = (100 \times 1/20) = 5$$

Hence, the required rate of interest is 5% per annum

**Question 18:** *In what time will Rs.1000 become Rs.1331 at 10% per annum compounded annually:*

a. 2 Years

b. 3 Years

c. 4 Years

d. 5 Years

**Explanation:**

$\rightarrow$  Principal = Rs.1000;

$\rightarrow$  Amount = Rs.1331;

$\rightarrow$  Rate = Rs.10%p.a.

$\rightarrow$  Let the time be n years then,

$$\rightarrow 1000(1+10/100)^n = 1331$$

$$\rightarrow (11/10)^n = 1331/1000$$

$$\rightarrow (11/10)^3 = 1331/1000$$

$\rightarrow$  So answer is 3 years

**Question 19:** *Certain sum becomes 3 times itself at compound interest in 10 years. In how many years it becomes 9 times?*

a. 25 years

b. 27 years

c. 30 years

d. 20 years

**Explanation:**

$$\rightarrow P(1 + R/100)^{10} = 3P$$

$$\rightarrow P(1 + R/100)^{10} = 3$$

$$\rightarrow \text{Let } P(1 + R/100)^n = 9P$$

$$\rightarrow (1 + R/100)^n = 9$$

$$\rightarrow 3^2 = [(1 + R/100)^{10}]^2$$

$$\rightarrow (1 + R/100)^n = (1 + R/100)^{20}$$

→  $n = 20$  Years.

### 3.6 ASSIGNMENTS - COMPOUND INTEREST

1. Vijay deposited Rs. 8000 in a bank that pays 12% interest per annum compounded quarterly. Find the amount he will receive after 6 months.

- |                |                   |
|----------------|-------------------|
| a. Rs. 8785.30 | b. Rs. 9275.5     |
| c. Rs. 9300    | d. Rs. Rs. 8487.2 |

2. Find the rate of interest per annum if Rs. 10000 amounts to Rs. 17280 in 3 years. Consider compound interest yearly.

- |         |         |
|---------|---------|
| a. 25%  | b. 20 % |
| c. 15 % | d. 10 % |

3. Find the time in which Rs. 3300 will become Rs. 3399 at 6% per annum if the interest is compounded half yearly.

- |             |             |
|-------------|-------------|
| a. 3 months | b. 4 months |
| c. 2 months | d. 6 months |

4. A bank lent Rs. 1331 to a group of companies at CI and got Rs. 1728 after three years. Find the interest rate charged if the interest is compounded annually.

- |           |           |
|-----------|-----------|
| a. 9.09 % | b. 9.08 % |
| c. 9.07 % | d. 9.1 %  |

5. What will be the principal if the interest is compounded at the rate of 10% per annum for 2 years is Rs. 420.

- |             |             |
|-------------|-------------|
| a. Rs. 2200 | b. Rs. 1800 |
| c. Rs. 2000 | d. Rs. 1600 |

6. Find the CI on Rs. 1000 at the rate of 20% per annum for 18 months when interest is compounded half yearly.

- |            |            |
|------------|------------|
| a. Rs. 300 | b. Rs. 331 |
| c. Rs. 362 | d. Rs. 393 |

7. Rs. 1694 is repaid after 2 years at CI. Find which of the following is the value of the principle and the interest rate

- |                   |                  |
|-------------------|------------------|
| a. Rs. 1400, 10 % | b. Rs. 1400, 8 % |
| c. Rs. 1500, 10 % | d. Rs. 1500, 8 % |

**8.** The sum of Rs. 2100 is lent at CI of 5% p.a. for 2 yrs. What will be the amount after 2 years?

a. Rs. 2351.25

b. Rs. 2300

c. Rs. 2315.25

d. Rs. 2400

**9.** What will be the difference between CI (compounded annually) and SI at 5% per annum for 2 yrs on the amount of Rs. 2000?

a. Rs. 8

b. Rs. 6

c. Rs. 5

d. Rs. 7

**10.** The difference between CI and SI on a certain sum of money at 20% per annum for 3 years is Rs. 48. What is the sum of money?

a. Rs. 375

b. Rs. 350

c. Rs. 400

d. Rs. 325

### ***3.7.1 Answer Keys- Compound Interest***

<b>Sr. No</b>	<b>Ans</b>	<b>Sr. No</b>	<b>Ans</b>
<b>1</b>	d	<b>6</b>	b
<b>2</b>	b	<b>7</b>	a
<b>3</b>	d	<b>8</b>	c
<b>4</b>	a	<b>9</b>	c
<b>5</b>	c	<b>10</b>	a



## UNIT - 4

### RATIO AND PROPORTION, MIXTURES AND ALLIGATION

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#### Learning Objective

**After going through this unit, you will be able to understand:**

- Basic Calculation of Ratios
  - Understand of Variation and Proportion
  - Understand Mixtures and Allegations
  - Understand approaches of solving questions on Ratio and Proportion
- 

#### 4.1 Theoretical Framework of Ratio and Proportion

##### What is Ratio and Proportion?

The definition of ratio and proportion is described here in this section. Both concepts are an important part of Mathematics. In real life also, you may find a lot of examples such as the rate of speed (distance/time) or price (rupees/meter) of a material etc., where the concept of the ratio is highlighted.

Proportion is an equation which defines that the two given ratios are equivalent to each other. For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of 500km for 5 hours. Such as  $100\text{km/hr.} = 500\text{km}/5\text{hrs} = 100\text{km/hr.}$

let us now learn Maths ratio and proportion concept one by one.



## Definition of Ratio

In certain situations, the comparison of two quantities by the method of division is very efficient. We can say that the comparison or simplified form of two quantities of the same kind is referred to as ratio. This relation gives us how many times one quantity is equal to the other quantity. In simple words, the ratio is the number which can be used to express one quantity as a fraction of the other ones.

The two numbers in a ratio can only be compared when they have the same unit. We make use of ratios to compare two things. **The sign used to denote a ratio is ‘:’.**

A ratio can be written as a fraction, say  $\frac{2}{5}$ , or can be represented by using “to”, as “2 to 5.” We happen to see various comparisons or say ratios in our daily life.

### Key Points to Remember:

- The ratio should exist between the quantities of the same kind
- While comparing two things, the units should be similar
- There should be significant order of terms
- The comparison of two ratios can be performed, if the ratios are equivalent like the fractions

## Definition of Proportion

Proportion is an equation which defines that the two given ratios are equivalent to each other. In other words, the proportion states the equality of the two fractions or the ratios. In proportion, if two sets of given numbers are increasing or decreasing in the same ratio, then the ratios are said to be directly proportional to each other.

For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of 500km for 5 hours. Such as  $100\text{km/hr} = 500\text{km}/5\text{hrs} = 100\text{km/hr}$ .

Ratio and proportions are said to be faces of the same coin. When two ratios are equal in value, then they are said to be in **proportion**. In simple words, it compares two ratios. **Proportions are denoted by the symbol ‘::’ or ‘=’.**

## Continued Proportion

Consider two ratios to be **a: b** and **c: d**.

Then in order to find the continued proportion for the two given ratio terms, we convert the means to a single term/number. This would, in general, be the LCM of means.

For the given ratio, the LCM of b & c will be bc.

Thus, multiplying the first ratio by c and second ratio by b, we have

First ratio-  $ca:bc$

Second ratio-  $bc:bd$

Thus, the continued proportion can be written in the form of  **$ca:bc:bd$**

## Ratio and Proportion Formula

Now, let us learn the Maths ratio and proportion formulas here.

### Ratio Formula

Assume that, we have two quantities or two numbers or two entities and we have to find the ratio of these two, then the formula for ratio is defined as;

$$a:b \Rightarrow a/b$$

where a and b could be any two quantities.

Here, “a” is called the first term or **antecedent**, and “b” is called the second term or **consequent**.

Example: In ratio 4:9, is represented by  $4/9$ , where 4 is antecedent and 9 is consequent.

If we multiply and divide each term of ratio by the same number (non-zero), it doesn't affect the ratio.

$$\text{Example: } 4:9 = 8:18 = 12:27$$

### Proportion Formula

Now, let us assume that, in proportion, the two ratios are  **$a:b$**  &  **$c:d$** . The two terms ‘b’ and ‘c’ are called ‘**means or mean term**,’ whereas the terms ‘a’ and ‘d’ are known as ‘**extremes or extreme terms**.’

$$a/b = c/d \text{ or } a : b :: c : d$$

---

Let us consider one more example of a number of students in a classroom. Our first ratio of the number of girls to boys is 3:5 and that of the other is 4:8, then the proportion can be written as

$$3:5 :: 4:8 \text{ or } 3/5 = 4/8$$

Here 3 & 8 are the extremes, while 5 & 4 are the means.

**Note:** The ratio value does not affect when the same non-zero number is multiplied or divided on each term.

#### **4. 1.1 VARIATION**

When two or more quantities are so related that if one of them is changed the other is changed with respect to it, the relation is called variation.

##### *Direct Variation*

One quantity is said to vary directly as another when the two quantities are so related that if one of them be increased (or decreased) the other increases (or decreases) in the same ratio i.e. if  $x$  varies directly as  $y$ , then,  $x \propto y$  or  $x = ky$ , where  $k$  is any constant.

##### *Inverse Variation*

One quantity is said to vary inversely as another when the two quantities are so related that if one of them be increased (or decreased) the other is decreased (or increased) in the same ratio. i.e. if  $x$  varies inversely as  $y$ , then  $x \propto 1/y$  or  $x = k/y$ , where  $k$  is any constant.

Note: The sign  $\propto$  is called the sign of variation.

#### **4.1.2 PARTNERSHIPS**

If two or more persons invest their money (capital) in a joint business, their association is called partners

Partnership is of two kinds: Simple and Compound

1. Simple Partnership: If the capitals of several partners are invested for the same period, then it is called simple partnership.

2. Compound Partnership: If the capitals are invested for different periods, then it is called compound partnership.

A partner who simply invests money but does not attend to the business is called a sleeping partner. One who invests money as well as attends business is a working partner.

Generally, the gain or loss is divided among the partners in a partnership on the basis of following rules:

1. In a partnership, the gain or loss is distributed among the partners in the ratio of their capital investments when the investments of all the partners are for the same period.

## 4.2 SOLVED EXAMPLES - RATIO, PROPORTION AND VARIATION

**Question 1:** A mixture contains alcohol and water in the ratio 4 : 3. If 5 liters of water is added to the mixture, the ratio becomes 4: 5. Find the quantity of alcohol in the given mixture.

- a. 10
- b. 12
- c. 15
- d. 18

**Explanation:**

- Let the quantity of alcohol and water be  $4x$  liters and  $3x$  liters respectively
- $4x/(3x+5) = \frac{4}{5}$
- $20x = 4(3x+5)$
- $8x = 20$
- $x = 2.5$
- Quantity of alcohol =  $(4 \times 2.5)$  liters = 10 liters.

**Question 2:** A bag contains 50 P, 25 P and 10 P coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.

- a. 360, 160, 200
- b. 160, 360, 200
- c. 200, 360, 160
- d. 200, 160, 300

**Explanation:**

- Let ratio be  $x$ . Hence no. of coins be  $5x$ ,  $9x$ ,  $4x$  respectively
- Now given total amount = Rs.206
- $(0.50)(5x) + (0.25)(9x) + (0.10)(4x) = 206$
- we get  $x = 40$
- No. of 50p coins = 200
- No. of 25p coins = 360
- No. of 10p coins = 160

**Question 3:** Find the third proportional to 9 and 12?

- a. 9
- b. 108
- c. 16
- d. 48

***Explanation***

→ Formula = Third proportional =  $(b \times b)/a$

→  $A = 9$  and  $B = 12$

→  $(12 \times 12)/9 = 144/9 = 16$

***Question 4: In a certain room, there are 28 women and 21 men. What is the ratio of men to women? What is the ratio of women to the total number of people?***

a. 5:4

b. 4:7

c. 6:3

d. 8:7

***Explanation:***

→ Men : women =  $21 : 28 = 3:4$

→ Women : total number of people =  $28 : 49 = 4 : 7$

***Question 5: In a bag, there are a certain number of toy-blocks with alphabets A, B, C and D written on them. The ratio of blocks A:B:C:D is in the ratio 4:7:3:1. If the number of 'A' blocks is 50 more than the number of 'C' blocks, what is the number of 'B' blocks?***

a. 650

b. 480

c. 578

d. 350

***Explanation:***

→ Let the number of the blocks A,B,C,D be  $4x$ ,  $7x$ ,  $3x$  and  $1x$  respectively

→  $4x = 3x + 50$

→  $x = 50$ . So the number of 'B' blocks is  $7 \times 50 = 350$ .

***Question 6: Arrange the following ratios in descending order. 2 : 3, 3 : 4, 5 : 6, 1 : 5***

a.  $5 : 6 > 3 : 4 > 2 : 3 > 1 : 5$

b.  $5 : 6 > 3 : 4 < 2 : 3 > 1 : 5$

c.  $5 : 6 < 3 : 4 > 2 : 3 < 1 : 5$

d.  $5 : 6 < 3 : 4 < 2 : 3 < 1 : 5$

***Explanation:***

→ Given ratios are  $2/3$ ,  $3/4$ ,  $5/6$ ,  $1/5$

→ The L.C.M. of 3, 4, 6, 5 is  $2 \times 2 \times 3 \times 5 = 60$

→ Now,  $2/3 = (2 \times 20)/(3 \times 20) = 40/60$

→  $3/4 = (3 \times 15)/(4 \times 15) = 45/60$

→  $5/6 = (5 \times 10)/(6 \times 10) = 50/60$

→  $1/5 = (1 \times 12)/(5 \times 12) = 12/60$

→ Clearly,  $50/60 > 45/60 > 40/60 > 12/60$

→ Therefore,  $5/6 > 3/4 > 2/3 > 1/5$

→ So,  $5 : 6 > 3 : 4 > 2 : 3 > 1 : 5$

**Question 7: Find the fourth proportional to 2.4, 4.6 and 7.6?**

a. 14

b. 14.657

c. 15.56

d. 14.56

**Explanation:**

→ Formula = Fourth proportional =  $(b \times c)/a$

→  $A = 2.4$ ,  $B = 4.6$  and  $C = 7.6$

→  $(4.6 \times 7.6)/2.4 = 14.56$

**Question 8: If  $a:b=1:2$  and  $b:c=3:4$  find  $a:b:c$ ?**

a. 3:8:6

b. 3:12:6

c. 3:6:8

d. 1:2:4

**Explanation:**

→  $a:b = 1:2$ ,  $b:c = 3:4$

→ ( $a = 1 \times 3 = 3$ ,  $b = 2 \times 3 = 6$  and  $c = 2 \times 4 = 8$ )

→ ( $a = a \times b$ ,  $b = b \times b$  and  $c = b \times c$ )

→  $a:b:c = 3:6:8$

**Question 9: The length and width of a rectangle are in the ratio 3:5. If its perimeter is 48 cm, find its length and width**

a. 9,15

b. 6,18

c. 5,14

d. 8,20

**Explanation:**

- From the given ratio, length =  $3x$  and width =  $5x$
- Perimeter = 48
- $2(l+w) = 48$
- $2(3x + 5x) = 48$
- $8x = 24$  and  $x = 3$
- Therefore, Length =  $3(3) = 9$  cm and Width =  $5(3) = 15$  cm.

**Question 10: Given ratio are**

**$a:b = 2:3$**

**$b:c = 5:2$**

**$c:d = 1:4$**

**Find  $a:b:c$ .**

a.  $2:3:6$

b.  $4:5:6$

c.  $10:15:6$

d.  $1:5:7$

**Explanation:**

- Multiplying the first ratio by 5, second by 3 and third by 6, we have
- $a:b = 10: 15$
- $b:c = 15 : 6$
- $c:d = 6 : 24$
- In the ratio's above, all the mean terms are equal, thus
- $a:b:c:d = 10:15:6:24$

**Question 11: Rs. 120 are divided among A, B, C such that A's share is Rs. 20 more than B's and Rs. 20 less than C's. What is B's share**

a. Rs 10

b. Rs 20

c. Rs 24

d. Rs 28

**Explanation:**

- Let  $C = x$ . Then  $A = (x-20)$  and  $B = (x-40)$ .

→  $x + x - 20 + x - 40 = 120$  Or  $x=60$ .

→  $A:B:C = 40:20:60 = 2:1:3$ .

→ B's share = Rs.  $120 \times (1/6) = \text{Rs. } 20$

**Question 12:** *In a college, the ratio of the number of boys to girls is 8 : 5. If there are 200 girls, the total number of students in the college is*

- a. 420
- b. 520
- c. 620
- d. 720

**Explanation:**

→ Let the boy are  $8x$  and Girls are  $5x$

→  $5x = 200$ ;  $x = 40$

→ Total students =  $8x+5x = 13x = 13(40) = 520$

**Question 13:** *If the ratio of men to women at a party is 4:7, which of the following could be the number of people at the party?*

- a. 50
- b. 64
- c. 66
- d. 70

**Explanation:**

→  $4x+7x=11x$ . We know that the sum of the quantities, 11, represents a fraction of the total number of party goers, so our answer MUST be a multiple of 11.

→ The only multiple of 11 in our choices is C. 66.

**Question 14:** *In a group, the ratio of doctors to lawyers is 5:4. If the total number of people in the group is 72, what is the number of lawyers in the group?*

- a. 58
- b. 62
- c. 78
- d. 32

**Explanation:**

→ Let the number of doctors be  $5x$  and the number of lawyers be  $4x$ .

→ Then  $5x+4x = 72$



→  $x=8$ .

→ So the number of lawyers in the group is  $4 \times 8 = 32$

**Question 15:** In a mixture of 45 liters, the ratio of sugar solution to salt solution is 1:2. What is the amount of sugar solution to be added if the ratio has to be 2:1?

a. 5:4

b. 6:7

c. 1:5

d. 2:1

**Explanation:**

→ Number of liters of sugar solution in the mixture =  $(1 / (1+2)) \times 45 = 15$  liters.

→ So,  $45 - 15 = 30$  liters of salt solution is present in it.

→ Let the quantity of sugar solution to be added be  $x$  liters.

→ Setting up the proportion,

→ sugar solution / salt solution =  $(15+x)/30 = 2/1$  i.e.  $x = 45$ .

→ Therefore, 45 liters of sugar solution has to be added to bring it to the ratio 2:1.

**Question 16:** A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share?

a. Rs. 500

b. Rs. 1500

c. Rs. 2000

d. None of these

**Explanation:**

→ Let the shares of A, B, C and D be Rs.  $5x$ , Rs.  $2x$ , Rs.  $4x$  and Rs.  $3x$  respectively.

→ Then,  $4x - 3x = 1000$

→  $x = 1000$ .

→ B's share = Rs.  $2x = \text{Rs. } (2 \times 1000) = \text{Rs. } 2000$ .

**Question 17:** A sum of Rs.312 was divided among 100 boys and girls in such a way that the boy gets Rs.3.60 and each girl Rs.2.40 the number of girls is

a. 35

b. 40

c. 45

d. 50

**Explanation:**

→ Step (i)... Let x be the number of boys and y be the number of girls.

Given total number of boys and girls = 100 and  $x+y=100$

→ Step (ii) ....A boy gets Rs. 3.60 and a girl gets Rs. 2.40

The amount given to 100 boys and girls = Rs. 312

→  $3.6x+2.4y=312$  .....Step (iii)

→ Solving (i) and (ii)

→  $3.6x+3.6y=360$  multiplying by 3.6

→  $3.6x+2.4y-1.20yy=312-48=40$

→ The number of girls is 40.

**Question 18: Salaries of Ravi and Sumit are in the ratio 2:3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40:57. What is Sumit's salary?**

a. 38000

b. 46800

c. 36700

d. 50000

**Explanation:**

→ Let the original salaries of Ravi and Sumit be Rs. 2x and Rs. 3x respectively. Then,

→  $(2x+4000) / (3x+4000) = 40 / 57$

→  $57 \times (2x + 4000) = 40 \times (3x+4000)$

→  $6x = 68,000$   $3x = 34,000$

→ Sumit's present salary =  $(3x + 4000) = \text{Rs. } (34000 + 4000) = \text{Rs. } 38,000$

**Question 19: If  $a:b = 3:4$ ,  $b:c = 7:9$ ,  $c:d = 5:7$ , find  $a:d$ ?**

a. 5:12

b. 7:12

b. 3:11

d. 5:11

**Explanation:**

→  $a/d = (3/4) \times (7/9) \times (5/7) \Rightarrow 5/12$

**Question 20: Find the fourth proportional to 0.2, 0.12 and 0.3?**

a. 0.13

b. 0.18

c. 0.8

d. 0.15

***Explanation:***

Formula = Fourth proportional =  $(b \times c)/a$

A = 0.2 , B = 0.12 and C = 0.3

$(0.12 \times 0.3)/0.2$

$0.036/0.2 = 0.18$

***Question 21: There are three numbers such that one number is 20% more than a third number and the second number is 50% more than a first number. The ratio of two numbers is?***

a. 2 : 3

b. 3: 4

c. 4: 6

d. 3: 2

***Explanation:***

→ Let the third number be x.

→ Then, the first number =  $120x/100 = 6x/5$

→ second number =  $150/100 \times 6x/5 = 9x/5$

→ The ratio of two numbers is =  $6x/5 : 9x/5 = 2 : 3$

***Question 22: In a mixture 60 liters, the ratio of milk and water 2 : 1. If the this ratio is to be 1 : 2, then the quantity of water to be further added is***

a. 20 liters

b. 30 liters

c. 50 liters

d. 60 liters

***Explanation:***

→ Quantity of Milk =  $60 \times (2/3) = 40$  liters

→ Quantity of water =  $60 - 40 = 20$  liters

→ As per question we need to add water to get quantity 2:1

→  $40/(20+x) = 1/2$

→  $20 + x = 80$ . Hence, x = 60 liters

**Question 23:** *The number of candidates writing three different entrance exams is in the ratio 4:5:6. There is a proposal to increase these numbers of candidates by 40%, 60% and 85% respectively. What will be the ratio of increased numbers?*

- a. 14:15:16
- b. 12:15:19
- c. 13:19:21
- d. none of these

**Explanation:**

- Given ratio of number of candidates is 4:5:6
  - Let the number of candidates for 3 exams be 4k, 5k and 6k respectively.
  - After increasing, number of candidates become (140% of 4k), (160% of 5k) & (185% of 6k)
  - That is,  $(140 \times 4k)/100$ ,  $(160 \times 5k)/100$  and  $(185 \times 6k)/100 = 56k/10$ ,  $80k/10$  and  $111k/10$
  - Now, the required new ratio is:  $56k/100 : 80k/10 : 111k/10 = 56 : 80 : 111$
- Hence the answer is option d.

**Question 24:** *If  $A : B = 5 : 7$  and  $B : C = 6 : 11$ , then  $A : B : C$  is :*

- a. 55 : 77 : 66
- b. 30 : 42 : 77
- c. 35 : 49 : 42
- d. None of these

**Explanation:**

- $A : B = 5 : 7$ ,  $B : C = 6 : 11 = (6 \times 7/6) : (11 \times 7/6) = 7 : 77/6$
- $A : B : C = 5 : 7 : 77/6 = 30 : 42 : 77$ .

**Question 25:** If  $a:b=1:2$  and  $b:c=3:4$  find  $a:b:c$ ?

a. 3:8:6

b. 3:12:6

c. 3:6:8

d. 1:2:4

**Explanation:**

→  $a:b = 1:2$ ,  $b:c = 3:4$

→ ( $a = 1 \times 3 = 3$ ,  $b = 2 \times 3 = 6$  and  $c = 2 \times 4 = 8$ )

→ ( $a = a \times b$ ,  $b = b \times b$  and  $c = b \times c$ )

→  $a:b:c = 3:6:8$

### 4.3 ASSIGNMENT - RATIO AND PROPORTION

1. Find the fourth proportional to the numbers 2, 5, 4.

- a. 10
- b. 15
- c. 18
- d. 20

2. Find a third proportional to the numbers 2.5, 1.5.

- a. 0.8
- b. 0.9
- c. 0.7
- d. 1

3. The sum of two numbers is  $c$  and their quotient is  $p/q$ . Find the numbers.

- a.  $p, q$
- b.  $qc/p, pc/q$
- c.  $c/p+q, c/p+q$
- d.  $pc/p+q, qc/p+q$

4. Two numbers are in the ratio of 4:5 and the sum of these numbers is 27. Find the two numbers.

- a. 12, 15
- b. 15, 20
- c. 18, 30
- d. 9, 18

5. Three numbers are in the ratio 3:4:8 and the sum of these numbers are 975. Find the three numbers.

- a. 190,200,520
- b. 195,260,550
- c. 190,260,520
- d. 195,260,520

6. If  $A:B = 2:3$  and  $B:C = 4:5$  and  $C:D = 6:7$ , find  $A:D$ .

- a. 16:35
- b. 16:25
- c. 9:16
- d. 25:35

**7.** If  $A:B = 3:4$ ,  $B:C = 8:9$ , find  $A:B:C$ .

a. 6:8:13

b. 6:8:9

c. 3:5:9

d. 5:7:8

**8.** The ratio of two numbers is 5:9. If each number is decreased by 5, the ratio becomes 5:11. Find the numbers.

a. 15,30

b. 15,27

c. 20,27

d. 30,18

**9.** Find the number that must be subtracted from the terms of the ratio 5:6 to make it equal to 2:3.

a. 3

b. 4

c. 7

d. 2

**10.** In what ratio the two kinds of tea must be mixed together into one at Rs.9 per kg and another at Rs.15 per kg, so that mixture may cost Rs.10.2 per kg?

a. 4:1

b. 4:3

c. 3:1

d. 4:5

**11.** The ratio of present ages of Suresh and Mahesh is 7:5. If after six years their ages will be in the ratio of 4:3, the present age of Mahesh is

a. 32years

b. 36years

c. 30 years

d. None of these

**12.** Ravi's salary is 150% of Amit's salary. Amit's salary is 80% of Ram's salary. What is the ratio of Ram's salary to Ravi's salary?

a. 1:2

b. 2:3

c. 5:6

d. 6:5



**13.**  $\frac{1}{3}$ <sup>rd</sup> of the contents of a container evaporated on the 1<sup>st</sup> day,  $\frac{3}{4}$ <sup>th</sup> of the remaining evaporated on the second day. What part of the contents of the container is left at the end of the second day?

- a.  $\frac{1}{4}$   
c.  $\frac{1}{2}$
- b.  $\frac{1}{6}$   
d.  $\frac{1}{18}$

**14.** A bag contains Rs.216 in the form of one-rupee, 50-paise and 25-paise coins in the ratio of 2:3:4. The number of 50-paise coins is

- a. 96                                  b. 144  
c. 114                                 d. 141

**15.** If  $x$  varies directly as  $3y+1$  and  $x=9$  when  $y=1$ , then what is the value of  $x$  when  $y=5$ ?

- [illegible]

**16.** If the ratio of two numbers is  $(x - y) : (x + y)$  and if the value of this ratio is  $3 : 4$ , then what are the lowest possible integer values of  $x$  and  $y$ ?

- a. 7 and 1                      b. 10 and 1
- c. 1 and 3                      d. 5 and 2

**17.** Total number of students in a class is 'z' which is composed of 'x' girls and 'y' boys. What part of the class is composed of girls?

- a.  $x/(x+y)$
- b.  $zy/(x+y)$
- c.  $xz/xy$
- d.  $(zy + zx) / (x + y)$

**18.**  $\frac{1}{3}$ <sup>rd</sup> of the total girls and  $\frac{1}{2}$  of the total boys go to the canteen. How many total numbers of classmates go to the canteen?

- a.  $\frac{3}{7}$   
c.  $\frac{2}{5}$
- b.  $\frac{5}{6}$   
d. cannot be determined

**19.** The cost of an apartment is Rs. 2200 per sq. ft, whereas that of a row house is Rs.2800 per sq. ft. at the same location. The total costs of both are equal, find the ratio of both areas?

- a. 15:28
- b. 18:14
- c. 11:14
- d. 5:3

**20.** Two natural numbers whose sum is 72 cannot be in the ratio

- a. 1:3
- b. 3:5
- c. 1:2
- d. 3:4

**21.** Anu and Renu are partners in a business. Anu invests Rs 5000 for complete year & Renu invests Rs 2500 for 6 months. What is Renu's share if they earn Rs 240 as profit?

- a. 48
- b. 58
- c. 120
- d. 80

**22.** Three partners A, B and C invested Rs16000, Rs18000 and Rs23000 respectively in a business. How should they divide a profit of Rs19380?

- a. 8:9:23
- b. 8:18:23
- c. 16:18:23
- d. 16:9:23

**23.** A, B and C enter into partnership. A advances Rs12000 for 4 months, B Rs14000 for 8 months, and C Rs10000 for 10 months. They gain Rs5850 altogether. Find the ratio of share of each.

- a. 12:7:50
- b. 12:28:25
- c. 3:9:8
- d. 3:28:25

**24.** A starts a business with Rs. 20000. B joins him after 3 months with Rs40000. C puts a sum of Rs10000 in the business for 2 months only. At the end of the year the business gave a profit of Rs5000. How should the profit be divided among them?

- a. 12:18:1
- b. 6:9:1
- c. 12:9:2
- d. 6:18:1

[illegible]

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans
1	a	6	a	11	c	16	a	21	a
2	b	7	b	12	c	17	a	22	c
3	d	8	b	13	b	18	d	23	b
4	a	9	a	14	b	19	c	24	a
5	d	10	a	15	d	20	d	25	a

## 4.4 Mixtures and Alligations

In Mixtures, we study mixtures of two or more than two quantities that have different selling process or cost prices. Alligation is an important area of quantitative aptitude for various competitive exams. The rule of alligation enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price. This technique can be applied to any topic like mixtures, profit & loss, simple interest, time & distance, percentage, etc.

### Rule of Alligation

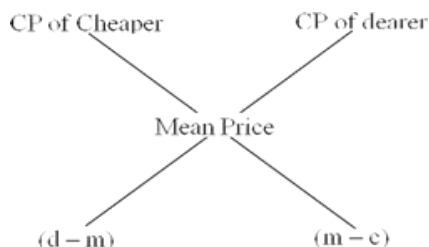
Alligation is a rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price. There are two types of methods used in alligation.

#### *Alligation Method:*

- It is a modified form of finding the weighted average. If 2 ingredients are mixed in a ratio and the cost price of the unit quantity of the mixture, called the Mean Price is given then,

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{CP of Cheaper}}$$

- The above formula can be represented with the help of diagram which is easier to understand. Here 'd' is the cost of dearer ingredient, 'm' is mean price and 'c' is the cost of cheaper ingredient.



$$\text{Hence } \frac{\text{Cheaper Quantity}}{\text{Dearer Quantity}} = \frac{(d - m)}{(m - c)}$$

**Question 1:** A 20 litre mixture of milk and water contains milk and water in the ratio 3 : 2. 10 liters of the mixture is removed and replaced with pure milk and the operation is repeated once more. At the end of the two removal and replacement, what is the ratio of milk and water in the resultant mixture?

- Cost of 1 litre of water = Rs.0 = cheaper quantity.
- Cost of 1 litre of juice = Rs.24 = dearer quantity.
- And, the mean price =  $m$  = Rs.20

→ Therefore, (Cheaper quantity) : (Dearer quantity) =  $(d - m) : (m - c) = 4:20 = 1:5$

→ Hence, the required answer is 1:5.

**Question 3:** A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

a.  $1/3$

b.  $1/4$

c.  $1/5$

d.  $1/7$

**Explanation:**

→ Suppose the vessel initially contains 8 liters of liquid.

→ Let  $x$  liters of this liquid be replaced with water.

→ Quantity of water in new mixture =  $3 - 3x/8 + x$  liters

→ Quantity of syrup in new mixture =  $5 - 5x/8$  liters

→  $3 - 3x/8 + x = 5 - 5x/8$

→  $5x + 24 = 40 - 5x$

→  $10x = 16$

→  $x = 8/5$ .

→ So, part of the mixture replaced =  $8/5 \times 1/8 = 1/5$ .

**Question 4:** A container contains 40 liters of milk. From this container 4 liters of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

a. 26 liters

b. 29.16 liters

c. 28 liters

d. 28.2 liters

**Explanation:**

→ milk contained by the container now

→  $= 40(1 - 4/40)^3$

→  $= 40(1 - 1/10)^3$

→  $= 40 \times 9/10 \times 9/10 \times 9/10$

$$\rightarrow = 29.16$$

**Question 5:** One type of liquid contains 25 % of benzene, the other contains 30% of benzene. A can is filled with 6 parts of the first liquid and 4 parts of the second liquid. Find the percentage of benzene in the new mixture.

- a. 28 %
- b. 25 %
- c. 30 %
- d. 27%

**Explanation:**

- $\rightarrow$  Let the percentage of benzene = X
- $\rightarrow (30 - X)/(X - 25) = 6/4 = 3/2$
- $\rightarrow 5X = 135$  or
- $\rightarrow X = 27$  so,
- $\rightarrow$  required percentage of benzene = 27 %

**Question 6:** How many kgs of Basmati rice costing Rs.42/kg should a shopkeeper mix with 25 kgs of ordinary rice costing Rs.24 per kg so that he makes a profit of 25% on selling the mixture at Rs.40/kg?

- a. 20.0 kgs
- b. 12.5 kgs
- c. 16.0 kgs
- d. 200.0 kgs

***Explanation:***

- Let the amount of Basmati rice being mixed be  $x$  kgs.
- As the trader makes 25% profit by selling the mixture at Rs.40/kg, his cost per kg of the mixture = Rs.32/kg.
- i.e.  $(x \times 42) + (25 \times 24) = 32(x + 25)$
- $42x + 600 = 32x + 800$
- $10x = 200$   
 $x = 20$  kgs.

***Question 7: In a mixture the ratio of alcohol to water is 5 : 4. If 5 liters more alcohol is added the ratio becomes 3 : 2. How many liters of alcohol was in the original mixture?***

- a. 15 liters
- b. 20 liters
- c. 22.50 liters
- d. 25 liters

***Explanation:***

- Let the quantity of alcohol and water in the mixture be  $5x$  and  $4x$  respectively.
- Thus, according to the problem,
- $10x + 10 = 12x$
- or,  $2x = 10$
- or,  $x = 5$
- Quantity of alcohol =  $5x = 5 \times 5 = 25$  liters

***Question 8: A can contain a mixture of two liquids A and B in the ratio 7: 5. When 9 liters of the mixture is drawn off and the can is filled with B, the ratio of A and B becomes 7: 9. How many liters of liquid A was contained by the can initially?***

- a. 10
- b. 20
- c. 21
- d. 25

***Explanation:***

- Suppose the can initially contain  $7x$  and  $5x$  liters of mixtures A and B respectively



- Quantity of A in mixture left =  $(7x - 7/12 \times 9) = (7x - 21/4)$  liters.
- Quantity of B in mixture left =  $(5x - 5/12 \times 9) = (5x - 15/4)$  liters.
- $(7x - 21/4) / [(5x - 15/4) + 9] = 7/9$
- $252x - 189 = 140x + 147$
- $x = 3$ .
- So, the can contain 21 liters of A.

**Question 9:** A housewife has 11 litre of Solution that contains sodium carbonate Solution and benzene Solution in the ratio 3 : 1. She adds 250 ml of 3 : 2 Solution of sodium carbonate and benzene Solution to it and then uses 250 ml of the combined mixture to make strong organic Solution How much of pure sodium carbonate Solution is she left with?

- a. 1000 ml
- b. 912.5 ml
- c. 750 ml
- d. 720 ml

**Explanation:**

- In a mixture of 1,000 ml sodium carbonate : benzene Solution = 3 : 1.
- Hence, sodium carbonate: 750 ml benzene Solution 250 ml
- A 250 ml of 3: 2 Solution contains 150 ml sodium carbonate Solution and 100ml benzene Solution.
- Total sodium carbonate Solution = 900 ml, total benzene Solution = 350 ml
- After using 250 ml to make strong organic Solution , sodium carbonate Solution used =  $250/1250 \times 900 = 180$  ml
- Pure sodium carbonate Solution left =  $900 - 180 = 720$ ml

**Question 10:** A vessel is filled with liquid, 3 parts of which are water and 5 parts of syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

- a.  $1/3$
- b.  $1/4$
- c.  $1/5$
- d.  $1/7$

**Explanation:**

- Suppose the vessel initially contains 8 liters of liquid.
- Let  $x$  liters of this liquid be replaced with water.
- Quantity of water in new mixture =  $(3-3x/8+x)$  liters.
- Quantity of syrup in new mixture =  $(5-5x/8)$  liters.
- $(3-3x/8+x) = (5-5x/8)$
- $5x + 24 = 40 - 5x$
- $10x = 16 \Rightarrow x = 8/5$
- So, part of the mixture replaced =  $(85 \times 18) = 1/5$ .

**Question 11:** Two varieties of wheat - A and B costing Rs. 9 per kg and Rs. 15 per kg were mixed in the ratio 3 : 7. If 5 kg of the mixture is sold at 25% profit, find the profit made?

- a. Rs. 13.50
- b. Rs. 14.50
- c. Rs. 15.50
- d. Rs. 16.50

**Explanation:**

- Let the quantities of A and B mixed be  $3x$  kg and  $7x$  kg. Cost of  $3x$  kg of A =  $9(3x) = \text{Rs. } 27x$
- Cost of  $7x$  kg of B =  $15(7x) = \text{Rs. } 105x$
- Cost of  $10x$  kg of the mixture =  $27x + 105x = \text{Rs. } 132x$
- Cost of 5 kg of the mixture =  $132x/10x (5) = \text{Rs. } 66$
- Profit made in selling 5 kg of the mixture =  $25/100$  (cost of 5 kg of the mixture) =  $25/100$   
 $\times 66 = \text{Rs. } 16.50$

**Question 12:** How many liters of water should be added to a 30 litre mixture of milk and water containing milk and water in the ratio of 7 : 3 such that the resultant mixture has 40% water in it?

- a. 5 liters
- b. 7 liters
- c. 10 liters
- d. None of these

**Explanation:**

- 30 liters of the mixture has milk and water in the ratio 7 : 3. i.e. the Solution has 21 liters of milk and 9 liters of water.

- When you add more water, the amount of milk in the mixture remains constant at 21 liters. In the first case, before addition of further water, 21 liters of milk accounts for 70% by volume.
- After water is added, the new mixture contains 60% milk and 40% water.
- Therefore, the 21 liters of milk accounts for 60% by volume.
- Hence, 100% volume =  $21/0.6=35$  liters.
- We started with 30 liters and ended up with 35 liters.
- Therefore, 5 liters of water was added.

***Question 13: A container contains 40 liters of milk. From this container 4 liters of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now contained by the container?***

- |              |                 |
|--------------|-----------------|
| a. 26 liters | b. 29.16 liters |
| c. 28 liters | d. 28.2 liters  |

***Explanation:***

- Suppose a container contains  $x$  units of a liquid from which  $y$  units are taken out and replaced by water.
- After  $n$  operations, a quantity of pure liquid =  $x (1-y/x)^n$  units.
- milk contained by the container now
- $40(1-4/40)^3 = 40(1-1/10)^3 = 40 \times 9/10 \times 9/10 \times 9/10$
- $4 \times 9 \times 9 \times 9 / 100 = 29.16$

***Question 14: How many liters of a 90% of concentrated acid needs to be mixed with a 75% Solution of concentrated acid to get a 30-liter Solution of 78% concentrated acid?***

- a. 8
- b. 9
- c. 7
- d. 6

***Explanation:***

- The concentration is given which is wrt 100, hence we can take  $x$  lt of 90% and  $(30-x)$  of 75%  
 $x \times 90 + (30-x) \times 75 = 30 \times 78$
- Hence the ans is 6 liters.

***Question 15: A zookeeper counted the heads of the animals in a zoo and found it to be 80. When he counted the legs of the animals he found it to be 260. If the zoo had either pigeons or horses, how many horses were there in the zoo? In the zoo, each horse had four legs and each pigeon had two legs.***

- a. 40
- b. 30
- c. 50
- d. 60

***Explanation:***

- Let the number of horses =  $x$
- Then the number of pigeons =  $80 - x$ .
- Each pigeon has 2 legs and each horse has 4 legs.
- Therefore, total number of legs =  $4x + 2(80 - x) = 260$
- $4x + 160 - 2x = 260$
- $2x = 100$
- $x = 50$ .

***Question 16: One type of liquid contains 25 % of benzene, the other contains 30% of benzene. A can is filled with 6 parts of the first liquid and 4 parts of the second liquid. Find the percentage of benzene in the new mixture.***

- a. 28 %
- b. 25 %
- c. 30 %
- d. 27%

***Explanation:***

- Let the percentage of benzene =  $X$
- $(30 - X)/(X - 25) = 6/4 = 3/2$
- $5X = 135$  or  $X = 27$
- so, required percentage of benzene = 27 %

***Question 17: A container contains 50 liters of milk. From this container, 10 liters of milk was taken out and replaced by water. This process is repeated one more time. How much milk is now left in the container?***

- a. 24
- b. 32
- c. 30
- d. 36

***Explanation:***

- Applying the Replacement Method: Amount of milk after 2 operations =  $[5(1 - 10/50)^2]$   
=  $50 \times 4/5 \times 4/5 = 32$  Liters

**Question 18:** Spirit and water in two container X and Y are in the ratio 5 : 2 and 7 : 6 respectively. In what ratio these mixtures be mixed to get a new mixture in container Z having water and spirit in the proportion 5 : 8?

- a. 5 : 6
- b. 7 : 9
- c. 3 : 4
- d. 4 : 3

**Explanation:**

- Let the C.P. of spirit be Rs. 1 per litre.
- Spirit in 1 litre mixture of X =  $\frac{5}{7}$  litre
- Spirit in 1 litre mixture of Y =  $\frac{7}{13}$  litre
- Spirit in 1 litre mixture of Z =  $\frac{8}{13}$  litre.

**Question 19:** 30 boys with an average weight of 60 kgs and 20 girls with an average weight of 40. Find the average weight of the whole class? Find the average weight of the whole class?

- a. 30
- b. 52
- c. 46
- d. 38

**Explanation:**

- To find the answer to the above question, one can use the formula for the weighted average.
- Suppose the question was framed a bit differently, the Average weight of boys is 60, the average weight of girls is 40 and average weight of the whole class is 52.

a. 15 liters                      b. 12 liters

c. 9 liters                         d. 6 liters

Let us assume the number of liters of the 90% purity solution = A  
and the number of liters of the 97% purity solution = B.

First Solution is 3x i.e.  $3 * 3 = 9$  Liters

a. 26.34 liters                      b. 27.36 liters

c. 28 liters                          d. 29.16 liters

→  $(40(1-4/40)^3) = [40*(9/10*9/10*9/10)] = 29.16$  liters

**Question 22:** From a container, 6 liters milk was drawn out and was replaced by water. Again 6 liters of mixture was drawn out and was replaced by the water. Thus the quantity of milk and water in the container after these two operations is 9:16. The quantity of mixture is:

- a. 15
- b. 16
- c. 25
- d. 31

**Explanation:**

- Let quantity of mixture be  $x$  liters. Suppose a container contains  $x$  units of liquid from which  $y$  units are taken out and replaced by Water. After  $n^{\text{th}}$  operations, the quantity of pure liquid =  $x(1-y/x)^n$  units, Where  $n$  = no of operations.
- So, Quantity of Milk =  $x(1-6/x)^2$
- Given that, Milk : Water = 9 : 16
- Milk : (Milk + Water) = 9 : (9+16)
- Milk : Mixture = 9 : 25
- Therefore,  $x(1-6/x)^2/x = 9/25$
- $x = 15$  liters

**Question 23:** For a car there are 5 tyres including one spare tyre(4+1). All tyres are equally used. If the total distance travelled by car is 40000km then what is the average distance travelled by each tyre?

- a. 10000
- b. 40000
- c. 32000
- d. 8000

**Explanation:**

- Total distance travelled by the car = 40000 km
- So total distance travelled by 4 wheels =  $4 \times 40000 = 160000$  as all tyres(4+1) are equally used so average distance travelled by each tyre =  $160000/5 = 32000$



#### 4.6 ASSIGNMENT - MIXTURES AND ALLIGATIONS

1. In what ratio the rice of inferior quality (price – Rs. 15/kg) should be mixed with the rice of superior quality (price – Rs. 25/ kg) so that the seller will gain Rs. 2 after selling the rice for Rs. 20/kg.

- |        |        |
|--------|--------|
| a. 5:3 | b. 7:5 |
| c. 7:3 | d. 7:6 |

2. Find the price of a particular type of wheat in Rs./kg if 10 kg of it is mixed with 20 kg of another type of wheat (price – Rs. 18/kg) so that the seller earns nothing after selling the mixed wheat for Rs. 15/kg.

- |                |              |
|----------------|--------------|
| a. Rs. 9.50/kg | b. Rs. 9/kg  |
| c. Rs. 10/kg   | d. Rs. 11/kg |

3. A milkman mixes 1 liter of water to 'x' liters of milk and sells the mixture for Rs. 20/liter. If the price of pure milk is Rs 25/liter then find the value of 'x'.

- |      |      |
|------|------|
| a. 5 | b. 3 |
| c. 2 | d. 4 |

4. The owner of a wine shop gains Rs. 25 after selling one bottle of the mixture of water and wine in the proportion 1:5 for Rs. 100. Find the price of one bottle of pure wine.

- |           |           |
|-----------|-----------|
| a. Rs. 80 | b. Rs. 90 |
| c. Rs. 85 | d. Rs. 95 |

5. A builder uses cement formed by mixing an 2 kg of inferior quality cement of price Rs. 80/kg to 3 kg of superior quality cement of price Rs. 120/kg. Find the price in Rs./kg of the cement that the builder uses should be considered for costing purposes.

- |               |               |
|---------------|---------------|
| a. Rs. 108/kg | b. Rs. 100/kg |
| c. Rs. 104/kg | d. Rs. 95/kg  |

6. A fruit seller mixes strawberries of two types in the ratio 1:3. The quantity of better quality strawberries is more in the mixture. He sells the mixture of strawberries for Rs. 100/kg and there by gains Rs. 20. Find the price of the strawberries of inferior quality in Rs./kg if the price of the strawberries of the superior quality is Rs. 90/kg.

- |              |              |
|--------------|--------------|
| a. Rs. 50/kg | b. Rs. 55/kg |
| c. Rs. 60/kg | d. Rs. 45/kg |

7. A tank contains 100 liters of petrol. 10% of petrol was replaced with diesel. This process was repeated four times. Find the amount of petrol remaining in the tank.

- a. 60 liters
- c. 70 liters

- b. 63 liters
- d. None of these

**8.** A reservoir contains 1000 liters of liquid A. Alternately 10% and 20% liters of liquid was replaced with water. Find the amount of liquid A left in the tank if such alternate replacements were carried out two times.

- a. 518.4 liters
- c. 750 liters

- b. 525 liters
- d. None of these

**9.** A vessel contains 100 liters of mixture of water and alcohol. The ratio of alcohol to water is 1:4. Another vessel contains 200 liters of mixture of water and alcohol. The ratio of water to alcohol is 1:4. The mixtures from both the vessels are poured into a third vessel. Find the ratio of alcohol to water in the third vessel.

- a. 3:4
- c. 3:2

- b. 2:3
- d. 1:3

**10.** A vessel contains liquid A and water in the ratio 4:1 Another vessel contains liquid A and water in the ratio 1:4 In what proportion the mixture from the first vessel should be mixed with the mixture from the second vessel so that the net ratio of the amount of liquid A is to amount of water becomes 7:3

- a. 4:1
- c. 3:1

- b. 5:1
- d. 3:2

**11.** A vessel contains 100 liters of pure milk. 20% of the liquid in the vessel is now replaced with water. This procedure was carried out three times. Find the amount of milk in the vessel.

- a. 55 liters
- c. 60 liters

- b. 50 liters
- d. None of these

**12.** The weight of an empty bottle is  $\frac{1}{6}$ th the weight of the bottle full of water. A certain percentage of the water is removed from the bottle full of water and the bottle was weighed. The weight of the bottle turned out to be the  $\frac{1}{3}$ rd the weight of the bottle full of water. Find the percentage of the water removed.

- a. 80%
- c. 70%

- b. 75%
- d. 85%

**13.** A vessel contains 'x' liters of pure acid. 10% of acid is replaced with water. This process was repeated once more. The difference in the quantity of acid in the vessel and the quantity of water in the vessel now is 62 liters. How much does the vessel hold?

- a. 110 liters
- c. 100 liters

- b. 125 liters
- d. 90 liters

**14.** A cup of coffee contains 1 part of pure milk and 3 parts of water (with dissolved coffee powder). How much part of the water (with dissolved coffee powder) must be replaced with milk so that the resulting mixture is half milk and half water?

- a. One-fourth
- b. One-third
- c. One-fifth
- d. One-sixth

**15.** 2 liters of water is added to 8 liters of 25% solution of alcohol in water. What is the strength of alcohol now?

- a. 22 %
- b. 19 %
- c. 21 %
- d. 20 %

**16.** In what proportion a salt at Rs. 3.2 per kg be mixed with salt at 2.9 per kg so that the mixture will be worth Rs. 3.08 per kg.

- a. 3:2
- b. 2:3
- c. 3:4
- d. 4:3

**17.** How many kg of tea selling at Rs. 5.2 a kg be mixed with tea selling at Rs. 4.4 a kg to make mixture of 15 kg. at Rs. 73.2?

- a. 7 kg
- b. 8 kg
- c. 9 kg
- d. 10 kg

**18.** A dealer mixes varieties of grains at Rs. 12 per kg and Rs. 30 per kg so that he can gain 10% by selling the resulting mixture at Rs.16.5 a kg. Find the proportion in which the grains are mixed?

- a. 5:2
- b. 5:1
- c. 2:3
- d. 3:2

**19.** A milk seller buys milk at Rs. 6 a liter and after adding water he sells it at Rs. 7.2 a liter and thus gains 60%. What is the percentage of water in the mixture?

- a. 26 %
- b. 24 %
- c. 23 %
- d. 25 %

**20.** What should be the proportion in which the water must be mixed with milk to gain  $33\frac{1}{3}\%$  by selling the mixture at cost price?

- a. 1:3
- b. 2:3
- c. 3:1
- d. 3:2

#### 4.6.1 ANSWER KEYS - MIXTURES AND ALLIGATIONS

Q. No.	Ans	Q. No.	Ans	Q. No.	Ans	Q. No.	Ans
1	c	6	a	11	d	16	a
2	b	7	d	12	a	17	c
3	d	8	a	13	c	18	b
4	b	9	c	14	b	19	d
5	c	10	b	15	d	20	a



## **UNIT - 5**

### **AVERAGES AND PROBLEMS ON AGES**

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#### **Learning Objective**

**After going through this unit, you will be able to understand:**

- Basic Calculation of Average/Mean, Mode and Median
  - Moderate questions on Averages, Alligations
  - Problems on Ages
  - Practice on Averages and Ages
- 

#### **5.1 Theoretical Framework of Averages**

So, in this section, we are going to provide the aspirants who are willing to train themselves in the Averages with the Basic Formulas. While cracking any of the problems, these formulae will come handy for you all. Therefore, try to check them and understand them Now, apply these fundamentals to some problems. Solved examples are given after the theory part. After feeling confident, give assignments to master the topic.

Well, below, you got two different formulas. And one is to calculate the Normal Average, while the other is to calculate the Average Speed.

**Formula 1:**

Average or mean: The Mean (Average) of a group of numbers is the sum of the numbers divided by the number of numbers:

**Average(A) / Arithmetic Mean** = (Sum of observations S / Number of observations)

**Formula****2:**

If the average of 'm' quantities is 'x' and the average of 'n' other quantities is 'y' then the

average of all of them put together is =  $\frac{mx + ny}{m+n}$

**Formula 3:** Average Speed =  $(2xy)/(x+y)$  kmph

**Formula 4:** If terms are in Arithmetic Progression:

- Average = Middle Term or (First Term + Last Term)/2

The sum of all the quantities of the same kind divided by their number is referred to as average of those quantities.

The average is also called Arithmetic Mean. Thus, the sum of all the quantities = Number of quantities x Average

**Geometric Mean:** Unlike arithmetic mean, the geometric mean takes into account the product of the numbers. For two numbers 'a' and 'b', the geometric mean is defined as:  $\sqrt{ab}$

For three numbers 'a', 'b' and 'c', the geometric mean is defined as:  $\sqrt[3]{abc}$

**Harmonic Mean:** The harmonic mean of two numbers 'a' and 'b' is defined as  $\frac{2ab}{a+b}$

*If A.M denotes the arithmetic mean, G.M denotes the geometric mean and H.M, the harmonic mean, then the relationship between the three is given by:*

$$A.M \times G.M = H.M^2$$

Their relationship can also be illustrated using the inequality:  $A \geq G \geq H$

**5.2 Mean, Median and Mode:** The "mean" is the "average" you're used to, where you add up all the numbers and then divide by the number of numbers. The "median" is the "middle" value in the list of numbers. To find the median, your numbers have to be listed in numerical order from smallest to largest, so you may have to rewrite your list before you can find the median. The "mode" is the value that occurs most often. If no number in the list is repeated, then there is no mode for the list.

**Example:** Find the mean, median, mode, and range for the following list of values:

13, 18, 13, 14, 13, 16, 14, 21, 13

The mean is the usual average, so we will add and then divide:

$$(13 + 18 + 13 + 14 + 13 + 16 + 14 + 21 + 13) \div 9 = 15$$

Note that the mean, in this case, isn't a value from the original list. This is a common result. You should not assume that your mean will be one of your original numbers.

The median is the middle value, so first I'll have to rewrite the list in numerical order:

13, 13, 13, 13, 14, 14, 16, 18, 21

There are nine numbers in the list, so the middle one will be the  $(9 + 1) \div 2 = 10 \div 2 = 5$ th number:

13, 13, 13, 13, 14, 14, 16, 18, 21

*So the median is 14.*

The mode is the number that is repeated more often than any other, so 13 is the mode.

The largest value in the list is 21, and the smallest is 13, so the range is  $21 - 13 = 8$ .



### 5.3 SOLVED EXAMPLES - AVERAGES

**Question 1:** *The average runs of a cricket player of 10 innings was 32. Find the total runs he scored in all 10 innings?*

- a. 760
- b. 180
- c. 385
- d. 320

**Explanation:** Average = Sum of Observations divided by Number of Observations

⇒ Hence,  $32 = x$  (to be found out)  $\div 10$

⇒ Solving this we get  $x = 32 \times 10$ , hence the answer is **320**.

**Question 2:** *Find the average of the first 10 whole numbers.*

- a. 7.5
- b. 4.5
- c. 10
- d. 5

**Explanation:** As we know, whole numbers start from 0. First 10 whole numbers would be 0,1,2,3..9.

⇒  $0 + 1 + 2 + 3 + \dots + 9$  divided by 10.

⇒  $45/10 = 4.5$

**Question 3:** *Average of all prime numbers between 30 to 50.*

- a. 37
- b. 37.8
- c. 39
- d. 39.8

**Explanation:**

⇒ Prime numbers between 30 and 50 are: 31, 37, 41, 43, 47

⇒ Average of prime numbers between 30 to 50 will be

⇒  $((31+37+41+43+47)/5) = 199/5 = 39.8$

**Question 4:** *The captain of a cricket team of 11 members is 26 years old and the wicketkeeper is 3 years older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. What is the average age of the team?*

- a. 23 years
- b. 24 years
- c. 25 years
- d. None of these

**Explanation:**

⇒ Let the average age of the whole team by x years.

$$\Rightarrow 11x - (26 + 29) = 9(x - 1)$$

$$\Rightarrow 11x - 9x = 46$$

$$\Rightarrow 2x = 46$$

$$\Rightarrow x = 23.$$

⇒ So, the average age of the team is **23 years**.

**Question 5: Average of 10 matches is 32, How many runs one should score to increase his average by 4 runs.**

a. 70

b. 76

c. 78

d. 80

**Explanation:**

⇒ Average after 11 innings should be 36

⇒ So, Required score =  $(11 * 36) - (10 * 32) = 396 - 320 = \mathbf{76 \text{ Runs}}$

**Question 6: There are 36 boys and 44 girls in a class. The average score of boys is 40 points and girls is 35 points. Then what will be the average score of the class?**

a. 37.25 Points

b. 28.95 points

c. 35.5 points

d. 40.04 points

**Explanation:**

⇒ Average score of 36 boys = 40 points

⇒ Total score of 36 boys =  $36 \times 40 = 1440$     ⇒ Average score of 44 girls = 35 points

⇒ Total score of 44 girls =  $35 \times 44 = 1540$

⇒ Total score of  $(36+44)$  80 Students =  $1440 + 1540 = 2980$  points

⇒ Average score of the whole class =  $(2980 / 80) = 37.25$

⇒ Hence the required answer is **37.25 points**.

**Question 7: The mean weight of a group of seven boys is 56 kg. The individual weights (in kg) of six of them are 52, 57, 55, 60, 59 and 55. Find the weight of the seventh boy.**

a. 64

b. 54

c. 88

d. 28

***Explanation:***

- $\Rightarrow$  Mean weight of 7 boys = 56 kg.
- $\Rightarrow$  Total weight of 7 boys =  $(56 \times 7)$  kg = 392 kg.
- $\Rightarrow$  Total weight of 6 boys =  $(52 + 57 + 55 + 60 + 59 + 55)$  kg = 338 kg.
- $\Rightarrow$  Weight of the 7th boy = (total weight of 7 boys) - (total weight of 6 boys)
- $\Rightarrow (392 - 338)$  kg = 54 kg.
- $\Rightarrow$  Hence, the weight of the seventh boy is **54 kg**.

***Question 8: Average weight of 25 boys in a class is 48 kgs. The average weight of the class of 40 students is 45 kgs. What is the average weight of the 15 girls in the class?***

- a. 44 kgs
- b. 42 kgs
- c. 40 kgs
- d. 39 kgs

***Explanation:***

- $\Rightarrow$  Total weight of boys in the class =  $25 * 48 = 1200$  kgs
- $\Rightarrow$  Total weight of all students in the class =  $45 * 40 = 1800$  kgs
- $\Rightarrow$  Total weight of the girls in the class =  $1800 - 1200 = 600$  kgs
- $\Rightarrow$  Average weight of girls =  $600/15 = 40$  kgs

***Question 9: Average of 16 numbers is 8. If 2 is added to every number, the new average will be:***

- a. 13
- b. 9
- c. 10
- d. 8

***Explanation:***

- $\Rightarrow$  Average of 16 numbers = 8
- $\Rightarrow$  Thus, total of 16 numbers =  $16 * 8 = 128$
- $\Rightarrow$  When 2 is added to every number then total of 16 numbers increases by  $16 * 2 = 32$
- $\Rightarrow$  Thus, new total of 16 numbers =  $128 + 32 = 160$
- $\Rightarrow$  New average of 16 numbers =  $160 / 16 = 10$

***Question 10: In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach the target of 282 runs?***

- a. 6.25
- b. 5.5

c. 7.40

d. 50

**Explanation:**

$\Rightarrow$  Runs scored in the first 10 overs =  $10 \times 3.2 = 32$

$\Rightarrow$  Total runs = 282

$\Rightarrow$  Remaining runs to be scored =  $282 - 32 = 250$

$\Rightarrow$  Remaining overs = 40

$\Rightarrow$  Run rate needed =  $250/40 = 6.25$

**Question 11:** *The average weight of 15 girls in a group is 24 kg when a new girl included the average weight increases by 2. What is the weight of the new girl?*

a. 56 kgs

b. 28 kgs

c. 34 kgs

d. None of these

**Explanation :**

$\Rightarrow$  Average weight of 15 girls = 24 kg

$\Rightarrow$  Total weight of 15 girls =  $24 \times 15 = 360$  kg

$\Rightarrow$  Average after including a new girl =  $24 + 2 = 26$  kg

$\Rightarrow$  Total weight of 16 girls =  $26 \times 16 = 416$  kg

$\Rightarrow$  Weight of the new girl = Total weight of 16 girls - Total weight of 15 girls

$\Rightarrow 416 - 360 = 56$  kg. Hence the required answer is **56 kg**.

**Question 12:** *A batsman makes a score of 87 runs in the 17th match and thus increases his average by 3. Find his average after the 17th match?*

a. 36

b. 37

c. 38

d. 39

**Explanation:**

$\Rightarrow$  Let the average after 17th match is  $x$ , then the average before 17th match is  $x-3$

$\Rightarrow$  So  $16(x-3) + 87 = 17x$

$\Rightarrow x = 87 - 48 = 39$

**Question 13:** *The average weight of 8 person's increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person?*

a. 76kg

b. 76.5kg

c. 85kg

d. Data Inadequate

**Explanation:**

$\Rightarrow$  Total weight increased =  $(8 \times 2.5) \text{ kg} = 20 \text{ kg}$ .

$\Rightarrow$  Weight of new person =  $(65 + 20) \text{ kg} = 85 \text{ kg}$ .

**Question 14:** *The average of 50 numbers is 30. If two numbers, 35 and 40 are discarded, then the average of the remaining numbers is near?*

a. 28.32

b. 29.68

c. 28.78

d. 29.27

**Explanation:**

$\Rightarrow$  Total sum of 50 numbers =  $(50 \times 30) - (35 + 40)$

$\Rightarrow 1500 - 75 = 1425$

$\Rightarrow$  Average =  $(1425/48)$

**= 29.68**

**Question 15:** *The average of 11 numbers is 10.9. If the average of first six is 10.5 and that of the last six is 11.4 the sixth number is?*

a. 11

b. 11.3

c. 11.4

d. 11.5

**Explanation:**

$\Rightarrow$  1 to 11 =  $11 \times 10.9 = 119.9$

$\Rightarrow$  1 to 6 =  $6 \times 10.5 = 63$

$\Rightarrow$  6 to 11 =  $6 \times 11.4 = 68.4$

$\Rightarrow 63 + 68.4 = 131.4 - 119.9 = 11.5$  ( $6^{\text{th}}$  number is counted twice in total 131.4, so we removed it)

$\Rightarrow 6^{\text{th}}$  number = 11.5

**Question 16:** *A grocer has a sale of Rs 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs 6500?*

a. Rs. 4991

b. Rs. 5467

c. Rs. 5987

d. Rs. 6453

***Explanation:***

⇒ Total sale for 5 months = Rs. (6435 + 6927 + 6855 + 7230 + 6562) = Rs. 34009.

⇒ Required sale = Rs. [(6500 x 6) - 34009]

⇒           Rs.           (39000           -           34009)           =           **Rs.           4991.**

## 5.4 PROBLEMS ON AGES

### CONCEPT EXAMPLES

**1. A father was 4 times as old as his son 8 years ago. Eight years hence, father will be twice as old as his son. Find their present ages.**

- a. 30 years
- b. 50 years
- c. 40 years
- d. 60 years.

**Explanation:** Let son's age 8 years ago be  $x$  years.

$\Rightarrow$  Thus, father's age at that time  $= 4x$  years

$\Rightarrow$  After 8 years, son's age  $= (x+8) + 8 = (x+16)$  years

$\Rightarrow$  After 8 years, father's age  $= (4x+8)+8 = (4x+16)$  years

$\Rightarrow 2(x+16) = 4x + 16$  or  $x = 8$

$\Rightarrow$  The present age of the son  $= x+8 = 16$  years

$\Rightarrow$  The present age of the father  $= 4x+8 = 32+8 = \mathbf{40 \text{ years}}$

**2. A is twice as old as B was two years ago. If the difference in their ages be 2 years, find A's age.**

- a. 5 years
- b. 8 years
- c. 10 years
- d. 15 years

**Explanation:** Let B's age 2 years ago be  $x$  years A's present age  $= 2x$  years

$\Rightarrow$  Also  $2x - (x+2) = 2$  or  $x = 4$

$\Rightarrow$  A's age  $= 2 \times 4 = \mathbf{8 \text{ years}}$

**3. The age of a father 10 years ago was thrice the age of his son. Ten years hence, the father's age will be twice that of his son. The ratio of their present ages is:**

- a. 5:3
- b. 7:3
- c. 2:3
- d. 3:5

**Explanation:** Let the present ages of father and son be  $x$  and  $y$  years respectively.

$\Rightarrow$  Then  $(x-10) = 3(y-10)$  or  $3y-x = 20$  ----- (1)  
 $\Rightarrow$  and  $(x+10) = 2(y+10)$  or  $x-2y = 10$  ----- (2)  
 $\Rightarrow$  (1) + (2)  $y = 30$   
 $\Rightarrow$  Substituting  $y = 30$  in equation (1) we get  $x = 70$   
 $\Rightarrow$  Ratio of their ages =  $70 : 30$  or **7:3**

**4. Ratio of Ashok's age to Pradeep's age is equal to 4:3. Ashok will be 26 years old after 6 years. How old is Pradeep now?**

- |             |             |
|-------------|-------------|
| a. 10 years | b. 18 years |
| c. 20 years | d. 15 years |

**Explanation:** Let present ages of Ashok and Pradeep are  $4x$  and  $3x$  respectively.

$\Rightarrow$  Ashok's present age =  $(26-6) = 20$  years i.e.  $4x = 20$  and hence  $x = 5$

$\Rightarrow$  Pradeep's present age =  $3x$  i.e.  $3 \times 5 =$  **15 years**

**5. The ratio of the ages of father and son at present is 6:1. After 5 years the ratio will become 7:2. The present age of the son is:**

- |             |            |
|-------------|------------|
| a. 3 years  | b. 5 years |
| c. 10 years | d. 8 years |

**Explanation:** Let their present ages be  $6x$  and  $x$  years respectively.

$\Rightarrow$  Then,  $(6x + 5) / (x+5) = 7 / 2$

$\Rightarrow 2(6x+5) = 7(x+5)$

$\Rightarrow x = 5$ . Hence, Present age of the son = **5 years**

**6. Three years ago the average age of A and B was 18 years. With C joining them now, the average becomes 22 years. How old is C now?**

- |             |             |
|-------------|-------------|
| a. 24 years | b. 26 years |
| c. 30 years | d. 20 years |

**Explanation:**  $(A+B)$ 's total present age =  $(2 \times 18 + 3 + 3) = 42$  years

$\Rightarrow (A+B+C)$ 's total present age =  $22 \times 3 = 66$  years



$$\Rightarrow C's \text{ age} = 66 - 42 = \mathbf{24 \text{ years}}$$

**7. The sum of ages of 5 children born at intervals of 3 years each is 50 years. What is the age of the youngest child?**

- a. 4 years
- b. 10 years
- c. 8 years
- d. None of these

**Explanation:** Let the ages of children be  $x$ ,  $(x + 3)$ ,  $(x + 6)$ ,  $(x + 9)$  and  $(x + 12)$  years.

$$\Rightarrow \text{Then, } x + (x + 3) + (x + 6) + (x + 9) + (x + 12) = 50$$

$$\Rightarrow 5x = 20$$

$$\Rightarrow x = 4.$$

$\therefore$  Age of the youngest child  $= x = 4$  years.

**8. A father said to his son, "I was as old as you are at the present at the time of your birth". If the father's age is 38 years now, the son's age five years back was:**

- a. 14 years
- b. 19 years
- c. 33 years
- d. 38 years

**Explanation:** Let the son's present age be  $x$  years. Then,  $(38 - x) = x$

$$\Rightarrow 2x = 38.$$

$$\Rightarrow x = 19.$$

$\therefore$  Son's age 5 years back  $(19 - 5) = 14$  years.

**9. A is two years older than B who is twice as old as C. If the total of the ages of A, B and C be 27, then how old is B?**

- a. 7
- b. 8
- c. 9
- d. 10

**Explanation:** Let C's age be  $x$  years. Then, B's age  $= 2x$  years. A's age  $= (2x + 2)$  years.

$$\therefore (2x + 2) + 2x + x = 27$$

$$\Rightarrow 5x = 25$$

$$\Rightarrow x = 5.$$

Hence, B's age =  $2x = 10$  years.

**10. Present ages of Sameer and Anand are in the ratio of 5 : 4 respectively. Three years hence, the ratio of their ages will become 11 : 9 respectively. What is Anand's present age in years?**

- a. 24
- b. 27
- c. 40
- d. None of these

**Explanation:** Let the present ages of Sameer and Anand be  $5x$  years and  $4x$  years respectively.

$$\Rightarrow 9(5x + 3) = 11(4x + 3)$$

$$\Rightarrow 45x + 27 = 44x + 33$$

$$\Rightarrow 45x - 44x = 33 - 27$$

$$\Rightarrow x = 6.$$

$\therefore$  Anand's present age =  $4x = 24$  years.

## 5.5 ASSIGNMENTS - AVERAGES AND PROBLEMS ON AGES

1. The average marks obtained by 100 students are 75. Find the total marks.

- a. 9000
- b. 7500
- c. 800
- d. 750

2. A man paid 5 workers at the rate of Rs.200 each, 6 workers at the rate of Rs.250 each and 9 workers at the rate of Rs.300 each. What is the average pay of each worker?

- a. Rs.250
- b. Rs.255
- c. Rs.260
- d. Rs.265

3. The average temperature from Monday to Thursday is  $48^{\circ}$  and from Tuesday to Friday is  $52^{\circ}$ .

If the temperature on Monday is  $42^{\circ}$ , what is the temperature on Friday?

- a.  $52^{\circ}$
- b.  $55^{\circ}$
- c.  $58^{\circ}$
- d.  $51^{\circ}$

4. The average weight of 24 students of section A of a class is 58 kg and the average weight of 26 students of section B of the same class is 60.5 kg. Find the average approximate weight of all the 50 students of the class.

- a. 50 kg
- b. 59.3 kg
- c. 66 kg
- d. 66.5 kg

5. Find the average of first 81 natural numbers.

- a. 34
- b. 41
- c. 42
- d. 40.5

6. Find the average of squares of the natural numbers from 1 to 41.

- a. 800
- b. 770
- c. 682
- d. 581

7. The average weight of 25 persons is increased by 2 kg when one of them whose weight is 60 kg is replaced by a new person. What is the weight of new person?

- a. 80 kg
- b. 90 kg

c. 95.5 kg

d. 110 kg

**8.** A batsman has a certain average of runs for 12 innings. In the 13<sup>th</sup> inning, he scores 96 runs, thereby increasing his average by 5 runs. What is his average after the 13<sup>th</sup> innings?

a. 48

b. 64

c. 36

d. 72

**9.** A number 'a' equals 80% of the average of 5, 7, 14 and 'b'. If average of a and b is 26, find b.

a. 13

b. 26

c. 39

d. 52

**10.** In a mango farm, (a+2) trees yield an average of 60 mangoes per year, 'a' trees yield an average of 120 mangoes per year and (a-2) trees yield an average of 180 mangoes per year. If average yield per tree per year is 100, find a.

a. 2

b. 4

c. 8

d. 10

**11.** There is a sequence of 11 consecutive even numbers. The average of first 7 numbers is A.

What is the average of all the 11 numbers?

a. 7

b. 7+A

c. A

d. A+4

**12.** In a group of 120 members, one fifth are adult men, one-fourth are adult women and the rest are children. The average age of women is five-sixth of the average age of men. The average age of children is one-fourth of the average age of men. If the average age of men is 60 years, then the average age of 120 members is?

a. 35

b. 32.75

c. 38

d. 36.2

**13.** A person travels from P to Q at a speed of 40 km/hr and returns by increasing his speed by 50%. Find his average speed for both the trips.

a. 45 km/hr

b. 38 km/hr

c. 48 km/hr

d. 50 km/hr

**14.** If the mean of  $x, y, z$  is  $M$  and  $xy + yz + zx = 0$ , then what is the mean of  $x^2, y^2, z^2$ ?

- a.  $3M^2$
- b.  $6M^2$
- c.  $9M^2$
- d. Cannot be determined

**15.** The average of seven numbers is five. If the average of the first six of these numbers is four, what is the seventh number?

- d. 14
- b. 12
- c. 11
- d. 15

**16.** A student finds the average of 10 positive integers. Each integer contains two digits. By mistake, the boy interchanges the digits of one number say  $ba$  for  $ab$ . Due to this, the average becomes 1.8 less than the previous one. What was the difference of the two digits  $a$  and  $b$ ?

- a. 2
- b. 4
- c. 6
- d. 8

**17.** There are three classes A, B and C with 20, 30 and 50 students respectively. The average score in mathematics of students in class B is 16 more than that of the class C and the average score of those in class A is 2 more than the overall average of scores in class A, B and C. What is the difference between average score of class C and class A?

- a. 3.5
- b. 8.5
- c. Data Insufficient
- d. Cannot be determined

**18.** Suppose that there are 10 no.'s, out of them, the sets of four numbers are chosen and their average is computed. Will the average of these averages be equal to the average of the 10 numbers?

- a. Yes
- b. No
- c. Data Insufficient
- d. cannot be determined

**19.** It rained as much as on Wednesday as on all the other days of the week combined. If the average rainfall for the whole week was 3 cms; how much did it rain on Wednesday?

- a. 3 cm
- b. 8.5 cm
- c. 10.5 cm
- d. 11.8 cm

a. 50 years                      b. 40 years  
c. 55 years                      d. 48 years

Q. No.	Ans	Q. No.	Ans	Q. No.	Ans	Q. No.	Ans	Q. No.	Ans
1	b	2	c	3	c	4	b	5	b
6	d	7	d	8	c	9	c	10	b
11	d	12	b	13	c	14	a	15	c
16	a	17	b	18	a	19	c	20	b



## UNIT - 6

### Time and Work, Pipes and Cisterns

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#### Learning Objective

After going through this unit, you will be able to:

- Basic Calculation of Time and Work
  - Understand of Important concepts and formulas on time and work
  - Get acquainted with concepts of efficiency
  - Understand approaches of solving questions on time and work
- 

#### 6.1 Theoretical Framework of Time and Work

Before we plunge into time and work formulas, let's quickly go through the different terms that make up time and work formulas

- If A can do a piece of work in  $n$  days, then A's one day's work =  $1/n$
- If A's one day's work =  $1/n$ , then A can finish the work in  $n$  days.
- If A is thrice as good a workman B, then
  - The ratio of work done by A and B = 3:1
  - The ratio of time taken by A and B to finish work = 1:3
- Total work = No of days \* Efficiency.
- If a group of people are given salary for a job they do together, their individual salaries are in the ratio of their individual efficiencies if they work for the same number of days. Otherwise, salaries are divided in the ratio of units of work done.

#### Approach for Time and Work Problems



Now that we know the terms used in time and work concepts, let's establish the relationship between them. This will help you to tweak time and work formulas as per the need of the questions. **Questions on Time and Work Formulas usually fall in one of the following categories:**

1. The relation between days taken by individuals to complete a given work independently and to complete while working simultaneously or alternately.
2. Teams of men, women, children and time taken by the teams to complete work independently or while working simultaneously.

Most questions asked are standard types of questions. Also, the approach to solve questions is a very standard one. There are two approaches, which are exactly the same but seem to be different – an approach using the per day's work and approaching LCM. The two approaches are best explained through an example.

### **Time and work Example**

If A does a work in 10 days and B does the same work individually in 12 days, in how many days will the work be completed if they work simultaneously?

### **Working with different efficiencies | Time and Work Formulas**

Problems discussed in this section are based on time and work formulas we have discussed above.

Approach 1: Per day's work

If A can complete the work in 'x' days and B can complete the same work in 'y' days, when they work together, the time taken to complete the work is given below.

A can complete the work in 'x' days. So in one day, he will do  $1/x$  of the work. B can complete the work in 'y' days. So in one day, he will do  $1/y$  of the work. Total work done by both in one day =  $(1/x) + (1/y)$ .

Hence, the total time required to do the work =  $(xy)/(x + y)$  days.

**Answer:**

Since A completes the entire work in 10 days, A does  $1/10$ th of the work in 1 day.

Since B completes the entire work in 12 days, B does  $1/12$ th of the work in 1 day.

Working simultaneously, they do  $1/10 + 1/12 = 11/60$  of the work in 1 day. Thus total days taken by both working simultaneously =  $60/11$  days.

**Approach 2: LCM Method**

In this method, we assume the total amount of work to be completed as a finite divisible value and based on it, we proceed with the calculation. To make the calculation simpler, assume the total amount of work to be completed as the LCM of time taken by different people to complete the same piece of work.

**Answer:**

Let the amount of work be 60 units (LCM of 10 and 12). Since A does 60 units in 10 days, he does 6 units every day. Since B does 60 units in 12 days, he does 5 units every day. Working simultaneously, they do  $6 + 5 = 11$  units each day.

Thus to complete 60 units of work, they will take  $60/11$  days.

The two approaches are absolutely identical; it is just that in the earlier approach the work was assumed as 1 unit instead of 60 units.



**Explanation:**

- C's 1 day's work =  $\frac{1}{3} - \frac{1}{6} + \frac{1}{8} = \frac{1}{3} - \frac{7}{24} = \frac{1}{24}$ .
- A's wages : B's wages : C's wages =  $\frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1$ .
- C's share (for 3 days) = Rs.  $(3 \times \frac{1}{24} \times 3200) = \text{Rs. } 400$ .

**Question 4:** *A and B can do a piece of work in 4 days, while C and D can do the same work in 12 days. In how many days will A, B, C and D do it together?*

- a. 12 days
- b. 4 days
- c. 3 days
- d. 2 days

**Explanation:**

A, B, C and D will together take  $\frac{1}{4} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$  i.e. 3 days to complete the work.

**Question 5:** *A and B can together complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?*

- a. 4 days
- b. 5 days
- c. 6 days
- d. 7 days

**Explanation:**

- (A+B)'s 1 day work =  $\frac{1}{4}$
- A's 1-day work =  $\frac{1}{12}$
- B's 1 day work =  $(\frac{1}{4} - \frac{1}{12}) = \frac{3-1}{12} = \frac{1}{6}$ .
- So B alone can complete the work in 6 days

**Question 6:** *A completes a work in 12 days and B complete the same work in 24 days. If both of them work together, then the number of days required to complete the work will be*

- a. 8 days
- b. 5 days
- c. 6 days
- d. 7 days

**Explanation:**

- If A can complete a work in  $x$  days and B can complete the same work in  $y$  days, then, both of them together can complete the work in  $\frac{xy}{x+y}$  days
- Therefore, here, the required number of days =  $12 \times 24 / 36 = 8$  days.

**Question 7:** *A is twice as good a workman as B and is therefore able to finish a piece of work in 30 days less than b. In how many days they can complete the whole work; working together?*

- a. 10 Days
- b. 20 Days
- c. 30 Days
- d. 40 Days

**Explanation:**

- Ratio of times taken by A and B = 1 : 2.
- The time difference is (2 - 1) 1 day while B take 2 days and A takes 1 day.
- If difference of time is 1 day, B takes 2 days.
- If difference of time is 30 days, B takes  $2 \times 30 = 60$  days.
- So, A takes 30 days to do the work.
- A's 1 day's work =  $1/30$
- B's 1 day's work =  $1/60$
- (A + b.'s 1 day's work =  $1/30 + 1/60 = 1/20$
- A and B together can do the work in 20 days.

**Question 8:** *Anil and Babu can do a job in 8 days. Babu and Charan can do the same job in 12 days. Anil, Babu, and Charan can do the job in 6 days. In how many days Anil and Charan can complete the job?*

- a. 6 days
- b. 8 days
- c. 10 days
- d. 12 days

***Explanation:***

- Anil and Babu can do a job in 8 days
- Combined efficiency of Anil + Babu =  $100/8 = 12.5\% \dots(1)$
- Similarly, combined efficiency of Babu + Charan =  $100/12 = 8.33\% \dots(2)$
- Similarly, combined efficiency of Anil + Babu + Charan =  $100/6 = 16.66\% \dots(3)$
- So now, if we want to calculate the efficiency of Anil and Charan together, we need to first calculate the individual efficiency of Anil and Charan.  
So we first deduct equation 1 from equation 3 to get efficiency of Charan. Then we deduct equation 2 from equation 3 we will get efficiency of Anil.
- Let's calculate the efficiency of Charan =  $(\text{Anil} + \text{Babu} + \text{Charan}) - (\text{Anil} + \text{Babu}) = 16.66 - 12.5 = 4.16\%$   
In the same way, we calculate the efficiency of Anil =  $16.66 - 8.33 = 8.33\%$
- So, combined efficiency of Anil and Charan =  $8.33\% + 4.16\% \sim 12.5\%$   
They need =  $100/12.5 = 8$  days.  
So they can complete the job in 8 days.

***Question 9: Shush and Komal can do a job together in 7 days. Shush is  $7/4$  times as efficient as Komal. The same job can be done by Shush alone in:***

- a.  $49/4$  days
- b.  $49/3$  days
- c. 11 days
- d.  $28/3$  days

***Explanation:***

- Ratio of efficiency of Shush and Komal =  $1:7/4 = 4:7$
- As the time taken is inversely proportional to efficiency, therefore, if Komal takes  $7x$  days to complete work, Asha will take  $4x$  days.
- Therefore  $1/7x + 1/4x = 1/7 \Rightarrow (4+7)/28x = 1/7$   
 $\Rightarrow 28x = 11 \times 7$

→ Therefore  $x = \{(11 \times 7)/28\} = 11/4$

Shush will complete the work in  $4x = 4 (11/4) = 11$  days.

**Question 10:** *A and B together can complete a task in 20 days. B and C together can complete the same task in 30 days. A and C together can complete the same task in 30 days. What is the respective ratio of the number of days taken by A when completing the same task alone to the number of days taken by C when completing the same task alone?*

a. 1:3

b. 2:4

c. 3:1

d. 4:5

**Explanation:**

→ Efficiency of A and B =  $1/20$  per day = 5% per day \_\_\_\_\_1

→ Efficiency of B and C =  $1/30$  per day = 3.33% per day \_\_\_\_\_2

→ Efficiency of C and A =  $1/30$  per day = 3.33% per day \_\_\_\_\_3

→ Taking equation 2 and 3 together

→  $B + C = 3.33\%$  and  $C + A = 3.33\%$

→ C and 3.33% will be removed. Hence  $A = B$

→ Efficiency of  $A = B = 5\%/2 = 2.5\% = 1/40$

→ Efficiency of  $C = 3.33\% - 2.5\% = 0.833\% = 1/120$

→ A can do the job in 40 days and C can do the job in 120 days if they work alone.

Ratio of number of days in which A and C can complete the job 1:3.

**Question 11:** *A and B can do a piece of work in 3 days, B and C in 4 days, C and A in 6 days. How long will C take to do it?*

a. 18 days

b. 20 days

c. 24 days

d. 30 days

**Explanation:**

→  $2c = \frac{1}{4} + \frac{1}{6} - \frac{1}{3} = \frac{1}{12}$

→  $c = 1/24 \Rightarrow 24$  days

**Question 12:** *To finish a work, a sets aside half additional time than b. In the event that together they take 18 days to finish the work, what amount of time might B take isn't that right?*

- a. 30 days
- b. 35 days
- c. 40 days
- d. 45 days

**Explanation:**

- Suppose B takes x days.
- Then, A takes  $150x/100$  days , i.e.  $3x/2$  days
- $1/x + 2/3x = 1/18$
- $5/3x = 1/18$
- $3x = 90$
- $x = 30$
- Hence B takes 30 days.

**Question 13:** *Susan can type 10 pages in 5 minutes. Mary can type 5 pages in 10 minutes. Working together, how many pages can they type in 30 minutes?*

- a. 15
- b. 20
- c. 25
- d. 75

**Explanation:**

( $30/5=6$ ;  $6*10=60$ ; Susan will type 60 pages in 30 min.  $30/10=3$ ;  $5*3=15$ ; Mary will type 15 pages in 30 min.  $60+15=75$ )

**Question 14:** *Two men, A and B, run a 4 km race on a circular course of  $1/4$  km. If their speeds are in the ratio of 5:4, how often does the winner pass the other?*

- a. Once
- b. Four times
- c. Twice
- d. Thrice



**Explanation:**

- When A runs 5 rounds, B runs 4 rounds (ratio of speeds)
- A passes B each time A has run 5 rounds or  $5 \times \frac{1}{4} = \frac{5}{4}\text{km.} = 1 \frac{1}{4}\text{km}$
- $1 \frac{1}{4}\text{km}$  is contained in 4 km 3 times. Hence A Passes B thrice.

**Question 15:** *A can do a piece of work in 30 days while B alone can do it in 40 days. In how many days can A and B working together do it?*

- a.  $\frac{16}{17}$
- b.  $\frac{17}{17}$
- c.  $\frac{18}{17}$
- d.  $\frac{19}{17}$

**Explanation:**

- Therefore, A's one day's work =  $\frac{1}{30}$
- B's one day's work =  $\frac{1}{40}$
- (A+B)'s one day's work =  $\frac{1}{30} + \frac{1}{40}$
- $\frac{4+3}{120} = \frac{7}{120}$

Number of days required for A and B to finish the work =  $\frac{17}{120} = 17 \frac{1}{7}\text{days}$

**Question 16:** *Adam can do a job in 15 days, Eve can do the same job in 20 days. If they work together for 4 days on this job. What fraction of job is incomplete?*

- a.  $\frac{1}{4}$
- b.  $\frac{1}{10}$
- c.  $\frac{7}{15}$
- d.  $\frac{8}{15}$

**Explanation:**

- Adam can do  $\frac{1}{15}$  of the job per day
- Eve can do  $\frac{1}{20}$  of the job per day
- If they work together they can do  $\frac{7}{60}$  of the work together
- Remaining job  $1 - \frac{7}{60} = \frac{53}{60} = \frac{8}{15}$

**Question 17:** *A takes twice as much time as B or thrice as much time as C to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in*

- a. 4 days
- b. 8 days

c. 6 days

d. 12 days

**Explanation:**

→ Suppose A, B and C take  $x$ ,  $x/2$  and  $x/3$  days respectively to finish the work.

→ Then,  $1/x + 2/x + 3/x = 1/2$

→  $6/x = 1/2$ , so,  $x = 12$ .

→ So, B takes  $(12/2) = 6$  days to finish the work.

**Question 18:** Anand finishes a work in 7 days, Bittu finishes the same job in 8 days and Chandu in 6 days. They take turns to finish the work. Anand on the first day, Bittu on the second and Chandu on the third day and then Anand again and so on. On which day will the work get over?

a. 3

b. 7

c. 9

d. 6

**Explanation:**

→ In the 1st day Anand does  $1/7$ th of total work

→ Similarly, Bithu does  $1/8$ th work in the 2nd day

→ Hence at the end of 3 days, work done =  $1/7 + 1/8 + 1/6 = 73/168$

→ Remaining work =  $(168-73)/168 = 95/168$

→ again after 6 days of work, remaining work is =  $(95-73)/168 = 22/168$

and hence Anand completes the work on 7th day

**Question 19:** Painter A can paint a house in 16 days, and painter B can do the same work in 20 days. With the help of painter C, they paint the house in 8 days only. Then, Painter C alone can do this task in

a. 80 days

b. 85 days

c. 79 days

d. 76 days

**Explanation:**

→ A's 1 day's work =  $1/16$  and B's 1 day's work =  $1/20$

→  $(A + B + C)$ 's 1 day's work =  $1/10$

→ C's 1 day's work =  $\frac{1}{10} - \frac{1}{16} + \frac{1}{20} = \frac{1}{80}$ , So, C can paint the house in 80 days.

**Question 20:** A tank has two leakages. The first leakage alone can empty the tank in 9 minutes and the second alone would have done it in 6 minutes. If water leaks out at a constant rate, how long does it take both the leakage together to empty the tank?

- a. 3.1 min
- b. 3.6 min
- c. 3.5 min
- d. 4 min

**Explanation:**

→ 1 minute's work of both the leakage =  $\frac{1}{9} + \frac{1}{6} = \frac{5}{18}$

→ So, both the leakage will empty the tank in  $\frac{18}{5}$  min

**Question 21:** 12 men work 8 hours per day to complete the work in 10 days. To complete the same work in 8 days, working 15 hours a day, the number of men required

- a. 4 days
- b. 5 days
- c. 6 days
- d. 8 days

**Explanation:**

→ That is, 1 work done =  $12 \times 8 \times 10$

→ Then,  $12 \times 8 \times 10 = m \times 15 \times 8$

→ (i.e. No. of men required. =  $12 \times 8 \times 10 / 15 \times 10 = 8$  days.

**Question 22:** In how many days can the work be done by 9 men and 15 women?

I. 6 men and 5 women can complete the work in 6 days.

II. 3 men and 4 women can complete the work in 10 days.

III. 18 men and 15 women can complete the work in 2 days.

- a. III only
- b. All I, II and III
- c. Any two of the three
- d. Any one of the three

**Explanation:**

Clearly, any two of the three will give two equations in  $x$  and  $y$ , which can be solved simultaneously.

***Question 23: A is twice as good as workman as B and together they finish a piece of work in 18 days. In how many days will B alone finish the work.***

- |            |            |
|------------|------------|
| a. 27 days | b. 54 days |
| c. 56 days | d. 68 days |

***Explanation:***

As per question, A do twice the work was done by b.

So A:B = 2:1

Also (A+B) one day work =  $1/18$

→ To get days in which B will finish the work, lets calculate work done by B in 1 day =  
 $=(1/18 * 1/3) = 1/54$

→ [Please note we multiplied by  $1/3$  as per B share and total of ratio is  $1/3$ ]

→ So B will finish the work in 54 days

***Question 24: A can do a piece of work in 5 days and B do the same work in 6 days. If A and B work together they will complete the work?***

a.  $9(3/5)$  days

b.  $9(1/5)$  days

c.  $9(2/5)$  days

d.  $30/11$  days

***Explanation:***

→ A's one day work is  $=1/5$ .  $1/5 * 30 = 6$ ;  $5 + 6 = 11/30$

→ B's one day work is  $=1/6$ .  $1/6 * 30 = 5$

→ One day work of A and B is  $=11/30$ ; So they will complete the work in  $30/11$  days

***Question 25: A work which could be finished in 9 days was finished 3 days earlier after 10 more men joined. The number of men employed was?***

a. 18

b. 20

c. 22

d. 24

***Explanation:***

→ let x men could have finished the work in 9 days

→ Now, (x + 10) can finish the same work in 6 days

→  $x * 9 = (x + 10) 6$ .

→ Solving this  $x = 20$ .

**6.3 ASSIGNMENT - TIME AND WORK**

**1.** A pipe can fill a tank in 5 hours. Find the part of the tank filled in one hour.

- |                  |                   |
|------------------|-------------------|
| a. $\frac{1}{2}$ | b. $\frac{1}{5}$  |
| c. $\frac{1}{3}$ | d. $\frac{1}{10}$ |

**2.** A pipe can empty a tank in 12 hours. Find the part of the tank emptied in 4 hours.

- |                  |                   |
|------------------|-------------------|
| a. $\frac{1}{2}$ | b. $\frac{1}{12}$ |
| c. $\frac{1}{3}$ | d. $\frac{1}{4}$  |

**3.** A pipe can fill a tank in 8 hours and another can empty it in 16 hours. If both the pipes are opened simultaneously, find the time taken to fill the tank.

- |          |           |
|----------|-----------|
| a. 8 hrs | b. 16 hrs |
| c. 4 hrs | d. 24 hrs |

**4.** Two pipes P and Q can fill a tank separately in 20 minutes and 30 minutes. If both the pipes are opened simultaneously, find the time taken to fill the tank.

- |           |                  |
|-----------|------------------|
| a. 10 min | b. 11 min        |
| c. 12 min | d. none of these |

**5.** Two pipes can fill a tank in 8 hours and 12 hours respectively, while a third pipe can empty it in 6 hours. If all the pipes are opened simultaneously, find the time taken to fill the tank.

- |           |           |
|-----------|-----------|
| a. 26 hrs | b. 24 hrs |
| c. 20 hrs | d. 22 hrs |

**6.** Two pipes can fill a tank in 30 mins and 60 mins respectively, while a leakage can empty it in 45 minutes. If all the pipes are opened simultaneously, find the time taken to fill the tank.

- |           |           |
|-----------|-----------|
| a. 30 min | b. 60 min |
| c. 36 min | d. 42 min |

**7.** A leak at the bottom of the tank can empty the full tank in 6 hours. An inlet fills water at the rate of 4 liters per minute. When the tank is full, the inlet is opened and due to leak, the tank gets empty in 8 hours. What is the capacity of the tank?

- a. 5760 liters
- b. 6600 liters
- c. 4550 liters
- d. 5885 liters

**8.** Three fourths of a tank is full of water. If 5 liters of water are added to it, four-fifths of the tank gets full. What is the capacity of the tank?

- a. 120 liters
- b. 90 liters
- c. 100 liters
- d. 80 liters

**9.** Two pipes P and Q running together can fill a tank in 6 minutes. If Q takes 5 minutes more than P to fill the tank, then the time in which P and Q will fill the tank separately will be respectively

- a. (15, 20) min
- b. (15, 10) min
- c. (10, 15) min
- d. (20, 25) min

**10.** A tank with capacity L liters is empty. If water flows into the tank from Pipe P at the rate of P liters per minute and water is pumped out by a pipe Q at the rate of Q liters per minute, and  $P > Q$ , then in how many minutes will the tank be filled?

- a.  $L/(Q - P)$
- b.  $L (P - Q)$
- c.  $L/(P - Q)$
- d.  $60L/(P - Q)$

**11.** A person can complete a piece of work in 6 days and another person can complete the same work in 12 days. If both of them work together, find the time taken to complete the work.

- a. 6 days
- b. 4 days
- c. 8 days
- d. 9 days

**12.** Three persons can finish a piece of work in 10, 15 and 18 days respectively. If all of them work together, what is the time taken to finish the work?

- a. 5 days
- b. 4.5 days
- c. 4 days
- d. 6 days

**13.** A and B can finish a piece of work in 12 days, B and C in 15 days and A and C in 20 days. How long would each of them take separately to do the same work?

- a. 20, 40, 60 days
- b. 10, 20, 30 days
- c. 15, 20, 25 days
- d. 30, 20, 60 days

**14.** Sohan takes twice as much time as Mohan to complete a work and Rohan does it at the same time as Sohan and Mohan together. If all of them working together can finish the work in 6 days, then find the time taken by each of them separately to finish the work.

- a. 20, 38, 14 days
- b. 36, 18, 12 days
- c. 24, 42, 18 days
- d. none of these

**15.** 5 bulls and 2 men working together can do four times as much work as a bull and a man together. Find the ratio of the work done by a bull to a man.

- a. 1:2
- b. 2:1
- c. 1:3
- d. 3:1

**16.** Running at the same constant rate, 6 identical machines can produce a total of 270 breads per minute. At this rate, how many breads could 10 such machines produce in 4 minutes?

- a. 900
- b. 1800
- c. 2700
- d. 3600



**17.** A man who works in a farm is paid Rs. 56 for 35 hours a week. Up to 40 hrs, he is paid at the normal rate and on overtime, 1.5 times the normal. How many hrs did he work to get Rs. 88?

- a. 48
- b. 52
- c. 58
- d. 50

**18.** If 10 cats can kill 20 mice in 3 days hunting for 12 hours a day. Then in how many days can 24 cats kill 32 mice hunting for 4 hours a day?

- a. 4
- b. 6
- c. 8
- d. 10

**19.** A man and a half can build a wall and a half in a day and a half, then how many walls do six men build in six days?

- a. 6
- b. 12
- c. 18
- d. 24

**20.** A computer can perform 30 identical tasks in six hours. At that rate, what is the minimum number of computers that should be assigned to complete 80 of the tasks within 3 hours?

- a. 5
- b. 6
- c. 7
- d. 8

**21.** A can do a piece of work in 10 days and B alone do it in 12 days. how much time will both take to finish if both of them are given a similar job?

- a. 60/11 days
- b. 58/11 days
- c. 55/9 days
- d. 61/11 days

**22.** A and B can do a piece of work in 10 days, B and C in 12 days, C and A in 15 days. In how many days will they finish it, if they all work together?

- a. 9 days
- b. 8 days

c. 7 days

d. 10 days

**23.** A can do a piece of work in 15 days and B can finish it in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the work?

a.  $28/3$  days

b.  $25/3$  days

c.  $31/3$  days

d.  $29/3$  days

**24.** A can do work in 12 days. B is 60% more efficient than a. Find the number of days it takes B to do the same piece of work.

a.  $17/2$  days

b.  $19/2$  days

c.  $21/2$  days

d.  $15/2$  days

**25.** 3 men and 6 women finish a job in 9 days, while 2 men and 8 women finish it in 12 days. In how many days will 12 women working alone finish the same job?

a. 432 days

b. 436 days

c. 36 days

d. 38 days

**6.3.1 Answers Key - Time and Work**

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans
1	b	6	c	11	b	16	b	21	a
2	c	7	a	12	b	17	d	22	b
3	b	8	c	13	d	18	b	23	b
4	c	9	c	14	b	19	d	24	d
5	b	10	c	15	b	20	b	25	c



## UNIT - 7

### TIME SPEED AND DISTANCE, PROBLEMS ON TRAINS

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#### Learning Objective

After going through this unit, you will be able to:

- Basic Calculation of Speed, Distance and Time
  - Understand of Important concepts and formulas
  - Understand different concepts in TSD
  - Understand approaches of solving questions on TSD
- 

#### 7.1 Theoretical Framework of Time, Speed and Distance

##### Time, Speed and Distance

In this article, we will be discussing important concepts, shortcuts, examples and formula of Time, Speed and Distance. **Time Speed Distance Formulas are easy to remember if you understand the concepts behind them.** So, go through this article thoroughly.

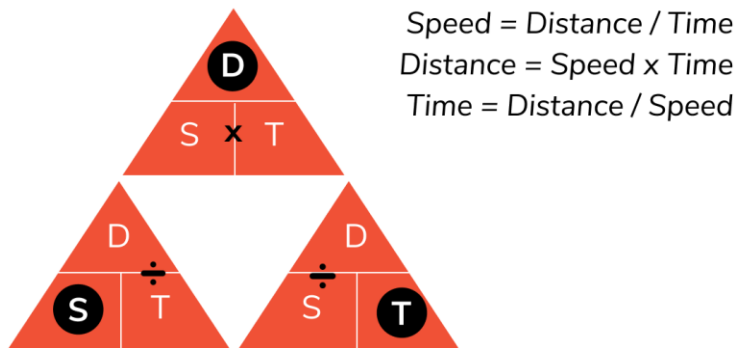
##### Time Speed Distance Triangle

Time (SI unit - seconds) and distance (SI unit - meters) are fundamental quantities of measurement. For a distance 'd' covered in a time duration 't', the average speed 's' or simply the speed is defined as the rate of covering the given distance or distance covered per unit time. Therefore, **the formula for Time Speed Distance is:**  $\text{Speed} = \text{Distance} / \text{Time}$

This means,

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \text{Distance} / \text{Speed}$$



**Note:** SI unit of speed is meters per second or m/sec. The other most commonly used unit of speed is kilometer per hour (kmph or km/h).

- To convert a **speed given in m/s into a speed in km/h**, multiply with 18/5.
- To convert a **speed given in km/h into a speed in m/s**, multiply with 5/18.

A simple way to remember the multiplication factor is to recall that particular speed, when expressed in km/h, is numerically larger than the same speed expressed in m/s. For other units like m/min or km/min, it is sufficient to remember that 1 km = 1000 m and 1 min = 60 sec.

### Relation between Time Speed Distance

From the above, the following can be concluded about the relations between Time Speed Distance

- If two bodies move at the same speed, the distances covered by them are (directly) proportionality to the times of travel. i.e. when s is constant,
- If two bodies move for the same time period, the distances covered by them are (directly) proportional to the speeds of travel. i.e. when t is constant,
- If two bodies move for the same distance, their times of travel are inversely proportional to the speeds of travel. i.e. when d is constant,

### Time Speed Distance Formula

Here are some important time speed distance formula you need to know to solve problems quickly.

### a) Average Speed

- When a body travels different distances at different speeds, the average speed is the amount of time taken to travel the total distance in total time, Therefore, the formula for Time Speed Distance is:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

- When the body travels at speeds of  $u$  and  $v$  units for equal distances, i.e. the distance segments are in the ratio 1:1, then

$$\text{Average speed} = \frac{2uv}{u + v}$$

### b) Relative Speed

Consider two bodies moving at speeds  $u$  and  $v$ .

- When they are moving in the same direction, the **relative speed between the two bodies** is the difference of their speeds, i.e.  $u - v$  if  $u > v$  or  $v - u$  if  $v > u$ .
- When they are moving in the opposite directions, the relative speed between the two bodies is the sum of their speeds, i.e.  $u + v$

### Types of problems in Time Speed Distance

#### 1) Key Rule for Ratio based Problems

- If two objects have their speeds in the ratio  $a:b$ , distance covered by the two objects in the same time will be in the ratio  $a:b$  and time taken by the two objects to cover same distance will be in the ratio  $b:a$ .
- The ratio concept can be applied only when one of the given distance, time or speed is constant.

#### 2) Overtaking, passing bodies

Problems may often feature bodies that move while other bodies may move or remain stationary. In such cases, the following points are to be noted.

- When two bodies pass each other (one body may be stationary), the speed of passing is equal to the relative speed between the two bodies.
- Objects like man, car, cycle, telegraph pole and tree are to be taken as point objects, with negligible length. When a train of length 'l' passes such an object, the distance covered while passing is equal to the length of the train 'l'.
- Objects like train, platform and bridge have length which needs to be taken into account when approaching the problem. When a train passes such an object of length 'p', the distance covered while passing is equal to the total length of the train and the object, i.e., 'l + p'. This holds true even when the second object is another train.

### 3) Travel and meeting

When two persons start from two points at the same time and travel towards each other, the time taken by each of them to reach the meeting point is the same. Hence, the distances covered by them from their respective starting points to the meeting point will be proportional to their respective speeds.

$$\frac{d_1}{d_2} = \frac{s_1}{s_2}$$

**For example**, assume A and B start simultaneously with speeds  $s_A$  and  $s_B$  respectively from points P and Q towards each other. They meet on the way at some point of time. From then onwards, A takes a time  $t_A$  and B takes a time  $t_B$  to reach Q and P respectively by continuing their onward travel. Then the following relationship is present between the speeds and times.

$$\frac{s_A}{s_B} = \sqrt{\frac{t_B}{t_A}} \text{ or } s_A^2 \times t_A = s_B^2 \times t_B$$

### 4) Boats and Streams based problems

The two quantities that come into the picture are the speed of the stream or river or current, and the speed of the boat or swimmer in still water.

- When the boat is moving against the current, it is said to move 'upstream'. Then the effective speed at which it moves is the difference between its speed in still water and the speed of the current.

- When the boat is moving with the current, it is said to move 'downstream'. Then the effective speed at which it moves is the sum of its speed in still water and the speed of the current.

**If  $u$  and  $v$  are the upstream and downstream speeds of the boat, then**

- Speed of the boat in still water =  $(u + v)/2$
- Speed of the stream =  $(v - u)/2$



## 7.2 SOLVED EXAMPLES - TIME SPEED AND DISTANCE

**Question 1:** A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?

- a. 3.6
- b. 7.2
- c. 8.4
- d. 10

**Explanation:**

- Speed =  $600 / 5 \times 60$  m/sec. = 2 m/sec.
- =  $2 \times 18/5$  km/hr;
- = 7.2 km/hr

**Question 2:** An aeroplane covers a certain distance at a speed of 240 kmph in 5 hours. To cover the same distance in 1 hours, it must travel at a speed of:

- a. 300 kmph
- b. 360 kmph
- c. 600 kmph
- d. 720 kmph

**Explanation:**

- Distance =  $(240 \times 5) = 1200$  km.
- Speed = Distance/Time
- Speed =  $1200 / (5/3)$  km/hr.
- Required speed =  $1200 \times 3/5$  km/hr = 720 km/hr.

**Question 3:** A car covers its journey at the speed of 80km/hr in 10hours. If the same distance is to be covered in 4 hours, by how much the speed of car will have to increase?

- a. 40km/hr
- b. 60km/hr
- c. 90km/hr
- d. 120km/hr

**Explanation:**

- Initial speed = 80km/hr
- Total distance =  $80 \times 10 = 800$  km
- new speed =  $800 / 4 = 200$  km/hr
- Increase in speed =  $200 - 80 = 120$  km/hr

**Question 4:** *An express train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point?*

- a. 6 hrs 27 min
- b. 6 hrs 24 min
- c. 6 hrs 21 min
- d. 6 hrs 30 min

**Explanation:**

- Time taken to cover 600 km =  $(600/100)$  hrs = 6 hrs.
- Number of stoppages =  $600/75 - 1 = 7$ .
- Total Time of stoppages =  $(3 \times 7)$  min = 21 min.
- Hence, total time taken = 6 hrs 21 min.

**Question 5:** *Two trains starting at the same time from 2 stations 200 km apart and going in opposite direction cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?*

- a. 11:9
- b. 7:3
- c. 18:4
- d. None of these

**Explanation:**

- In the same time, they cover 110 km and 90 km respectively.
- For the same time, speed and distance is inversely proportional.
- So ratio of their speed =  $110:90 = 11:9$

**Question 6:** *Walking at the rate of 4 kmph a man covers a certain distance in 2 hr 45 min. Running at a speed of 16.5 kmph the man will cover the same distance in.*

- a. 12 min
- b. 40 min
- c. 50 min
- d. 60 min

**Explanation**

- Distance = Speed  $\times$  time
- Here time = 2hr 45 min =  $11/4$  hr

- Distance =  $4 \times 11/4 = 11$  km
- New Speed = 16.5 kmph
- Therefore, time =  $D/S = 11/16.5 = 40$  min

**Question 7:** *Walking 6/7th of his usual speed, a man is 12 minutes too late. What is the usual time taken by him to cover that distance?*

- a. 1 hr 42 min
- b. 1 hr
- c. 2 hr
- d. 1 hr 12 min

**Explanation:**

- New speed = 6/7 of usual speed
- Speed and time are inversely proportional.
- Hence new time = 7/6 of usual time
- Hence, 7/6 of usual time - usual time = 12 minutes
- 1/6 of usual time = 12 minutes
- Therefore, usual time =  $12 \times 6 = 72$  minutes = 1 hour 12 minutes

**Question 8:** *The speed of a car increases by 2 kms after every one hour. If the distance travelling in the first one hour was 35 kms. what was the total distance travelled in 12 hours?*

- a. 456 km
- b. 482 km
- c. 558 km
- d. 552 km

**Explanation:**

- Total distance travelled in 12 hours =  $(35 + 37 + 39 + \dots \text{upto } 12 \text{ terms})$
- This is an A.P with first term,  $a = 35$ , number of terms,
- $n = 12$ ,  $d = 2$ .
- Required distance =  $12/2 [2 \times 35 + \{12 - 1\} \times 2]$
- $= 6(70 + 23)$
- $= 552$  kms

**Question 9:** *A runner can complete a 750 m race in two and a half minutes. Will he be able to beat another runner who runs at 17.95 km/hr?*

- a. Yes
- b. No

C. Insufficient Data

D. None

**Explanation:**

- We are given that the first runner can complete a 750 m race in 2 minutes and 30 seconds or 150 seconds.
- Speed of the first runner =  $750 / 150 = 5 \text{ m / sec}$
- We convert this speed to km/hr by multiplying it by 18/5.
- Speed of the first runner =  $18 \text{ km / hr}$
- Also, we are given that the speed of the second runner is 17.95 km/hr.
- Therefore, the first runner can beat the second runner.

**Question 10: Express 25 mps in kmph?**

a. 15 kmph

b. 99 kmph

c. 90 kmph

d. None

**Explanation:**

- $25 * 18/5 = 90 \text{ kmph}$

**Question 11: How long will a boy take to run round a square field of side 35 meters, If he runs at the rate of 9 km/hr?**

a. 50 sec

b. 52 sec

c. 54 sec

d. 56 sec

**Explanation:**

- Speed =  $9 \text{ km/hr} = 9 \times (5/18) \text{ m/sec} = 5/2 \text{ m/sec}$
- Distance =  $(35 \times 4) \text{ m} = 140 \text{ m}$ .
- Time taken =  $140 \times (2/5) \text{ sec} = 56 \text{ sec}$

**Question 12: A speed of a train is 20 meters per second. It can cross a pole in 10 seconds. What is the length of a train?**

a. 150 m

b. 250 m

c. 200 m

d. 300 m

**Explanation:**

- Length of train =  $20 \times 10 = 200 \text{ meters}$

**Question 13:** Nancy travelled a distance of 455 km by car in 10 hours. Find the speed of the car.

- a. 25.5 km/hr
- b. 45.5 km/hr
- c. 30 km/hr
- d. 65 km/hr

**Explanation:**

- Distance travelled by car = 455 km
- Time taken = 10 hours
- Therefore, speed = Distance/Time
- $455/10$  km/hr
- 45.5 km per hour

**Question 14:** A person goes from A to B at the speed of 40 kmph and comes back at the speed of 60 kmph. What is his average speed for the whole journey?

- A. 56 km/h
- B. 48 km/h
- C. 36 km/h
- D. 25 km/h

**Explanation:**

- Since the distance travelled on both sides is the same, we can use the formula of harmonic mean of speeds
- Average Speed:  $2xy/(x+y)$  where, x is the speed while going from A to B and y is the speed while coming back.
- So using this formula, we get the answer as 48 kmph.

**Question 15:** Wind blows 160 miles in 330min. for 80 miles how much time required

- a. 165
- b. 162
- c. 159
- d. 160

**Explanation:**

- 160 miles = 330 min
- 1 mile =  $330/160$
- 80 miles =  $(330 \times 80)/160 = 165$  min.

**Question 16:** Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

a. 9

b. 10

c. 12

d. 20

**Explanation:**

→ Due to stoppages, it covers 9 km less.

→ Time taken to cover 9 km =  $9/54 \times 60$  min = 10 min.

**Question 17:** A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?

a. 3.6

b. 7.2

c. 8.4

d. 10

**Explanation:**

→ Speed =  $(600/300)$  m/sec = 2 m/sec

→ Converting m/s to km/hr

→  $= 2 \times 8/15$  km/hr

→  $= 7.2$  km/hr

**Question 18:** A bus moves at a speed of 81 km/h. What is the speed of the bus in meters per second?

a. 22.65

b. 22.5

c. 28.5

d. 20.5

**Explanation:**

→ You can write,  $X = 81$  km/h

→ Therefore, speed in m/s (according to our formula.  $= X \times 5/18$

→  $(81 \times 5/18) = 22.5$  m/s

**Question 19:** A motorboat covers 25 km upstream and 39 km downstream by travelling at the same speed for 8 hours. On another occasion with the same speed, it covers 35 km upstream and 52 km downstream in 11 hours. What is the speed of the stream?

a. 5 km/hr

b. 4 km/hr

c. 3 km/hr

d. 2 km/hr

***Explanation:***

Each of the two periods of 8 hours and 11 hours consists of two component time periods, one upstream at a speed that is the difference in boat speed and stream speed, and the other downstream at a speed that is a sum of boat speed and stream speed

***Question 20: The ratio between the speeds of two trains is 7: 8. If the second train runs 400 km in 4 hours, then the speed of the first train is?***

- |               |               |
|---------------|---------------|
| a. 83.5 km/hr | b. 87.5 km/hr |
| c. 84.5 km/hr | d. 86.5 km/hr |

***Explanation:***

- Let the speeds of two trains be  $7x$  and  $8x$  km/hr.
- $8x = 400/4$
- $x = 12.5$  Km/hr
- So speed of first train is  $12.5 * 7 = 87.5$  km/hr

**Question 21:** A salesman sold twice as many pears in the afternoon than in the morning. If he sold 360 kilograms of pears that day, how many kilograms did he sell in the morning and how many in the afternoon?

- a. 240
- b. 200
- c. 150
- d. 360

**Explanation:**

- Let  $x$  be the number of kilograms he sold in the morning. Then in the afternoon he sold  $2x$  kilograms. So, the total is  $x+2x=3x$ .
- This must be equal to 360.
- $3x=360$ . Hence  $x=360/3$  and  $x=120$
- Therefore, the salesman sold 120 kg in the morning and  $2 \cdot 120=240$  kg in the afternoon.

**Question 22:** A car takes 6 hours to cover a distance of 540 Km. how much should the speed in Km/h be maintained to cover the same distance in  $3/2$ th of the previous time?

- a. 60 Km/h
- b. 62 Km/h
- c. 75 Km/h
- d. 77 Km/h

**Explanation**

- Time = 6
- Distance = 540
- $3/2$  of 6 hours =  $6 \times 3/2 = 9$  Hours
- Required speed =  $540/9 = 60$  Km/h

**Question 23:** A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minutes?

- a. 50 meter
- b. 110 meter
- c. None of above
- d. 100 meter

**Explanation:**

- Relative speed of the thief and policeman =  $(11 - 10)$  km/hr = 1 km/hr
- Distance covered in 6 minutes =  $1 \times 6 = 6$  km = 6000 meters
- So distance between them after 6 minutes =  $200 - 6000 = 100$  meters



**Question 24:** *Mary, Peter, and Lucy were picking chestnuts. Mary picked twice as many chestnuts than Peter. Lucy picked 2 kg more than Peter. Together the three of them picked 26 kg of chestnuts. How many kilograms did each of them pick?*

- a. 6,12,8
- b. 6,16,18
- c. 12,5,15
- d. 18, 5,25

**Explanation:**

→ Let  $x$  be the amount Peter picked. Then Mary and Lucy picked  $2x$  and  $x+2$ , respectively.

→ So  $x+2x+x+2=26$

→  $4x=24$

→  $x=6$

→ Therefore, Peter, Mary, and Lucy picked 6, 12, and 8 kg, respectively.

### **7.3 ASSIGNMENT - TIME SPEED AND DISTANCE**

**1.** How long does a train 100 meters long running at the rate of 40 km/hr take to cross a telegraphic pole?

- a. 9 hrs
- b. 9 min
- c. 9 sec
- d. 9.9 hrs

**2.** There is a big square shaped haunted palace in a jungle. A train 1000 meters long passes it with a speed of 120 km/hr in a minute. Find the area of the haunted palace.

- a. 1 sq.km
- b. 800 sq. m.
- c. 1.5 sq.km
- d. 1.2 sq.km

**3.** A car travels from P to Q with 15 km/hr and returns with 30 km/hr. What is the average speed of journey?

- a. 22.5 km/hr
- b. 20 km/hr
- c. 25 km/hr
- d. 10 km/hr

**4.** Three persons decided to go on a long tour and decided to drive the same distance of 100 km each. First person drove at the speed of 60 km/hr. After a break, the second person drove with the speed increased by 10 km/hr. Then at the end, the third person drove again with the speed increased by 10 km/hr. What is the average approximate speed of their journey?

- a. 75 km/hr
- b. 69 km/hr
- c. 100 km/hr
- d. none of these

**5.** A cycled from P to Q at 10 km/hr & returned at the rate of 9 km/hr. B cycled both ways at 12 km/hr in the whole journey. B took 10min less than A. find the distance between P & Q.

- a. 1.50 km
- b. 3.75 km
- c. 4.50 km
- d. 4.98 km

**6.** A person during his journey travels 40 minutes at a speed of 30 km/hr, another 50 minutes at 60 km/hr and last 1 hr at 30 km/hr. What is his average speed?

a. 38 km/hr

b. 40 km/hr

c. 60 km/hr

d. 35.7 km/hr

**7.** Sohan left his house at 7 am and walked at the rate of 2 km/hr to his college. But he saw that his college was closed and immediately he returned at the rate of 3 km/hr. If he had reached his house at 10 am, what is the distance of his college from his house?

a. 3 km

b. 5 km

c. 4 km

d. 3.6 km

**8.** By walking at  $\frac{3}{4}$ th of his usual speed Hemant is 6 minutes late to his office. What is his usual time to cover the distance?

a. 20 min

b. 18 min

c. 24 min

d. 1 hr

**9.** A train 145 meters long is running with a speed of 25 km/hr. In what time will it pass a man who is walking at 4 km/hr in the direction opposite to that of the train?

a. 10 sec

b. 8 sec

c. 9 sec

d. 18 sec

**10.** Two trains of lengths 100 meters and 120 meters are running in the same direction with speeds 30 km/hr and 40 km/hr respectively. In what time (approx.) will they pass each other?

a. 24 sec

b. 79 sec

c. 80 sec

d. 144 sec

**11.** Two persons are running in the same direction with the speeds of 3 km/hr and 6 km/hr. A truck comes running from behind and passes them in 9 and 10 seconds respectively. What is the speed of the truck?

a. 40 km/hr

b. 22 km/hr

c. 11 km/hr

d. 33 km/hr

**12.** A circular playground has an area 616 sq. meters. What time will it take for a runner to run around the circular ground at a speed of 22 km/hr?

a. 14 hrs

b. 14.4 sec

c. 16.2 sec

d. none of these

**13.** A train goes from P to Q in an hour and another train comes from Q to P in 1.5 hours. After what time will they meet each other?

a. 30 min

b. 36 min

c. 42 min

d. 48 min

**14.** Two trains starting at the same time from two stations 200 km apart, and going in opposite directions cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?

a. 9:20

b. 11:20

c. 11:9

d. 1:2

**15.** A cat can climb 6 meters of a round pole in one minute, but slips off 3 meters in the next minute. What time will it take to reach the top of a pole which is 21 meters long?

a. 14 min

b. 12 min

c. 11 min

d. none of these

**16.** A thief steals a car at 2.30 p.m. and drives it at 60 km/hr. The theft is discovered at 3 p.m. and the owner sets off in another car at 75 km/hr. When will he overtake the thief?

a. 4.30 p.m.

b. 4.45 p.m.

c. 5 p.m.

d. 5.15 p.m.

**17.** A and B start from home at 10am. They travel from the same point at 20 km/h and 40 km/h. There is a Junction T on their path. A turns left at the T junction at 12:00noon. B reaches T earlier, and turns right. Both of them continue to travel till 2pm. What is the distance between A and B at 2.00 pm?

a. 160 Km

b. 120 Km

c. 80 Km

d. 100 Km

**18.** Two cars travel simultaneously from the cities P and Q towards each other. Their speeds are 20 m/s and 25 m/s respectively. If  $d(P, Q) = 900$  km, find at what distance from Q will they meet?

a. 400 km

b. 500 km

c. 250 km

d. 200 km

**19.** Two trains travel toward each other on the same track, beginning 100 miles apart. One train travels at 40 miles per hour; the other travels at 60 miles an hour. A bird starts flight at the same location as the faster train, flying at a speed of 90 miles per hour. When it reaches the slower train, it turns around, flying the other direction at the same speed. When it reaches the faster train again, it turns around and so on. When the trains collide, how far will the bird have flown?

a. 50 miles

b. 72 miles

c. 118 miles

d. 90 miles

**20.** A nonstop bus to Mumbai overtakes an auto also moving towards Mumbai at 10 am. The bus reaches Mumbai at 12.30 pm and starts on the return journey after 1 hr. On the way back it meets the auto at 2 pm. At what time the auto will reach Mumbai?

- a. 2.30 pm
- b. 3 pm
- c. 3.30 pm
- d. cannot be determined

**21.** A train running at a speed of 90 km/hr passes a pole in 20 seconds. Find the length of the train.

- a. 100 m
- b. 500 m
- c. 400 m
- d. 300 m

**22.** A train meets an accident and travels at  $\frac{3}{4}$  of its original speed. Due to this, it is 20 minutes late. Find its original time beyond the point of accident.

- a. 40 min
- b. 50 min
- c. 60 min
- d. cannot be determined

**23.** A biker goes from his starting point to his destination with 20 km/hr, but returns with his speed increased by 50%. What is the average speed of his journey?

- a. 25 km/hr
- b. 30 km/hr
- c. 28 km/hr
- d. 24 km/hr

**24.** A jogger wants to save  $\frac{1}{4}$ th of his jogging time. By what percentage should he increase his speed?

- a. 25%
- b. 75%
- c. 33.33%
- d. 66.66%

**25.** Two cars travel simultaneously from the cities P and Q towards each other. Given that  $d(P, Q) = 100$  km. If they meet at a distance of 40 km from P, what is the ratio of their speeds?

a. 1:2

b. 2:3

c. 3:4

d. 4:5

### 7.3 ANSWER KEY - TIME SPEED AND DISTANCE

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans
1	c	6	b	11	d	16	c	21	b
2	a	7	d	12	b	17	a	22	c
3	b	8	b	13	b	18	b	23	d
4	b	9	d	14	c	19	d	24	c
5	b	10	b	15	c	20	b	25	b

## 7.4 PROBLEMS ON TRAINS

Problems on trains are very common in competitive exams. Various types of questions are asked on trains. Questions on trains are solved using the concept of time, speed and distance i.e. we use the formulas of time, speed and distance to solve questions on trains.

Given below is a list of some important points that need to be kept in mind while solving questions on trains.

### Important Points

- When two trains are going in the **same direction**, then their relative speed is the **difference between the two speeds**.
- When two trains are moving in the **opposite direction**, then their relative speed is the **sum of the two speeds**.
- When a train crosses a stationary man/ pole/ lamp post/ sign post- in all these cases, the object which the train crosses is stationary and the distance travelled is the length of the train.
- When it crosses a platform/ bridge- in these cases, the object which the train crosses are stationary and the distance travelled is the length of the train and the length of the object.
- When two trains are moving in the same direction, then their speed will be subtracted.
- When two trains are moving in opposite directions, then their speed will be added.
- In both the above cases, the total distance is the sum of the length of both the trains.
- When a train crosses a car/ bicycle/ a mobile man- in these cases, the relative speed between the train and the object is taken depending upon the direction of the movement of the other object relative to the train- and the distance travelled is the length of the train.



## 7.5 SOLVED EXAMPLES - PROBLEMS ON TRAINS

**Question 1:** *A train running at the speed of 60 km/hr crosses a pole in 9 seconds. What is the length of the train?*

- a. 120 metres
- b. 180 meters
- c. 150 meters
- d. 324 meters

**Explanation:**

- Speed =  $60 \times \frac{5}{18}$  m/sec =  $\frac{50}{3}$  m/sec.
- Length of the train = (Speed x Time).
- Length of the train =  $\frac{50}{3} \times 9$  m
- = 150 m.

**Question 2:** *A 400m long train is running at 72 Kmph. how much time will it take to cross an electric pole?*

- a. 15sec
- b. 20sec
- c. 19sec
- d. 21sec

**Explanation:**

- Distance = Speed × Time
- If we convert Kmph into Mpsec multiply by  $\frac{5}{18}$ ,
- $400 = 72 \text{ Kmph} \times \text{time}$
- $400 = 72 \times \frac{5}{18} \times \text{time}$ ; Time = 20Sec

**Question 3:** *A 180m long train is running at 54 Kmph. how much time will it take to cross a platform of 120m long?*

- a. 20sec
- b. 22sec
- c. 19sec
- d. 18sec

**Explanation:**

- $D = S \times T$
- $180 = 54 \text{ Kmph} \times \text{Time}$
- $180 = 54 \times 5/18 \times \text{Time}$
- $\text{Time} = 20 \text{ Sec}$

**Question 4:** A train of length 110 meters is running at a speed of 60 kmph. In what time, it will pass a man who is running at 6 kmph in the direction opposite to that in which the train is going?

- a. 10
- b. 8
- c. 4
- d. 6

**Explanation:**

- Distance = 110 m
- Relative speed =  $60 + 6 = 66 \text{ kmph}$  (Since both the train and the man are in moving in opposite direction) =  $(66 \times 5/18) \text{ m/sec} = 55/3 \text{ m/sec}$
- Time taken to pass the man =  $(100 \times 3/55) = 6 \text{ sec}$

**Question 5:** A train travelling at 60 kmph crosses a man in 6 seconds. What is the length of the train?

- a. 100 meters
- b. 150 meters
- c. 200 meters
- d. 250 meters

**Explanation:**

- Speed in m/sec =  $60 \times (5/18) = 50/3 \text{ m/sec}$
- Time taken to cross the man = 6 secs
- Therefore, Distance =  $(50/3) \times 6 = 100 \text{ metres}$  (i.e. the length of the train)

**Question 6:** A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platform?

- a. 120m
- b. 240m

c. 280m

d. 420m

**Explanation:**

- Speed =  $(54 \times \frac{5}{18})$  m/sec = 15 m/sec
- Length of the train =  $(15 \times 20)$  m = 300 m.
- Let the length of the platform be x meters.
- then,  $x + 300 / 36 = 15$
- $x + 300 = 540$ ;  $x = 240$ m

**Question 7 : If the speed of a train is 20 m/sec, find the speed of the train in kmph.**

a. 84 kmph

b. 72 kmph

c. 68 kmph

d. 55 kmph

**Explanation:**

- Speed = Distance / Time
- Speed = 20 m/sec
- Speed =  $20 \times \frac{18}{5}$  m/sec
- Speed = 72 kmph

**Question 8: A train is 100 meters long and is running at the speed of 30 km per hour. Find the time it will take to pass a man standing at a crossing.**

a. 12 seconds

b. 16 seconds

c. 14 seconds

d. 10 seconds

**Explanation:**

- As we need to get answer in seconds, so never forget to convert speed into meter per second.
- Speed = 30 km/hr =  $30 \times \frac{5}{18}$  m/sec  
=  $\frac{25}{3}$  m/sec
- Distance = length of train = 100 meter
- Required time = 12 seconds

**Question 9:** A train of length 250 m runs at a speed of 70 km/hr. What will be the time taken to cross any stationary object standing at the railway station?

- a. 17.23 sec
- b. 20 sec
- c. 9.5 sec
- d. 12.86 sec

**Explanation:**

- Length of train = 250 m, speed of train = 70 km/hr
- Length of train is always considered as distance, and hence here distance = 250 m
- 1) First convert speed of km/hr into m/s
- Speed of train =  $70 \times \frac{5}{18} = 19.44$  m/s
- 2) We know that,  
Speed = Distance/Time  
Time taken to cross stationary object =  $250/19.44$   
Time taken to cross stationary object = 12.86 sec

**Question 10:** A train running at the speed of 60 km/hr crosses a pole in 9 seconds. What is the length of the train?

- a. 120 metres
- b. 180 meters
- c. 324 meters
- d. 150 meters

**Explanation:**  $60 \times \frac{8}{15} \times 9 = 150$  meters

**Question 11:** A man's speed with the current is 15 km/hr and the speed of the current is 2.5 km/hr. The man's speed against the current is :

- a. 9.5 km/hr
- b. 10 km/hr
- c. 10.5 km/hr
- d. 11 km/hr

**Explanation:**

- Man's rate in still water =  $(15 - 2.5)$  km/hr = 12.5 km/hr.
- Therefore, Man's rate against the current =  $(12.5 - 2.5) = 10$  km/hr.

**Question 12:** A train of length 150 meters takes 40.5 seconds to cross a tunnel of length 300 meters. What is the speed of the train in km/hr?

- a. 13.33
- b. 40
- c. 26.67
- d. 66.67

**Explanation:**

- Speed =  $150 + 300/40.5$  m/sec
- =  $450/40.5 \times 18/5$  km/hr
- = 40 km/hr.

**Question 13:** Two trains are moving in opposite directions at 60 km/hr and 90 km/hr. Their lengths are 1.10 km and 0.9 km respectively. The time is taken by the slower train to cross the faster train in seconds is:

- a. 48 sec
- b. 52 sec
- c. 58 sec
- d. 66 sec

**Explanation:**

- Relative speed =  $(60 + 90)$  km/hr
- =  $[150 \times 5/18]$  m/sec =  $[125 / 3]$  m/sec.
- Distance covered =  $(1.10 + 0.9)$  km = 2 km = 2000 m.
- Required time =  $[2000 \times 3/125]$  sec = 48 sec.

**Question 14:** A train 200 m long is moving with the speed of 50km per hour. Find the time taken to pass a tree standing near the railway track.

- a.  $14 \frac{2}{5}$  seconds
- b. 15 seconds
- c.  $16 \frac{1}{2}$  seconds
- d. 17 seconds

**Explanation:**

- Speed of the train = 50km/hr =  $50 \times 5/18$  m/sec =  $250/18$  m/sec
- Length of the train = 200 m
- Time taken to cross the tree =  $200 / (250/18) = 200 \times 18/250 = 72/5 = 14.4$  seconds

→ Hence the answer is  $14 \frac{2}{5}$  seconds.

**Question 15:** *A train running at the speed of 60 km/hr crosses a pole in 9 seconds. What is the length of the train?*

a. 120m

d. 150m

c. 160m

d. 185m

**Explanation:**

→ Time is taken to cross a pole = time taken to cover a distance equal to its own length.

→ Speed of the train in m/s =  $60 \times \frac{5}{18} = \frac{50}{3}$  m/s.

→ Length of the train = Speed of the train x Time taken to cross the pole =  $\frac{50}{3} \times 9$

→ Length of the train = 150m.

**Question 16:** A train running at the speed of 60 km/hr crosses a pole in 9 seconds. Find the length of the train:

- a. 150 meter
- b. 145 meter
- c. 140 meter
- d. 135 meter

**Explanation:**

- Speed =  $60 \times \frac{5}{18}$  m/sec =  $\frac{50}{3}$  m/sec
- Length of Train(Distance) = Speed \* Time
- $\frac{50}{3} \times 9 = 150$  meters

**Question 17:** Two trains are running in opposite directions in the same speed. The length of each train is 120 meter. If they cross each other in 12 seconds, the speed of each train (in km/hr) is

- a. 42
- b. 36
- c. 28
- d. 20

**Explanation:**

- Distance covered =  $120 + 120 = 240$  m
- Time = 12 s
- Let the speed of each train = v. Then relative speed =  $v + v = 2v$
- $2v = \text{distance/time} = 240/12 = 20$  m/s
- Speed of each train =  $v = 20/2 = 10$  m/s
- $= 10 \times \frac{36}{10}$  km/hr = 36 km/hr

**Question 18:** A 400m long train is running at 72 Kmph. how much time it will take to cross an electric pole?

- a. 15sec
- b. 20sec
- c. 19sec
- d. 21sec

**Explanation:**

- Formula: Distance = Speed  $\times$  Time
- If we convert Kmph into Mpsec multiply by 5/18,
- $400 = 72 \text{ Kmph} \times \text{time}$
- $400 = 72 \times 5/18 \times \text{time}$
- Time = 20Sec

**Question 19:** The distance between two places A and B is 570 km. A train starts from A at 50 kmph at 6 am and another starts from B at 80 kmph at 7 am towards each other. At what time will they meet?

- a. 11 am
- b. 3 pm
- c. 8 am
- d. 10 am

**Explanation:**

- Let the two trains meet at a distance x km from place A
- The time required by the train starting from A to cover x is  $x/50$  hr
- Time taken by the other train starting from B to cover  $(570 - x)$  km =  $(570-x)/80$ , But the first train has started 1 hr early. So, it has travelled 50 km in this 1 hr.
- Therefore,  $x/50 - 1 = (570-x)/80$
- On Solving,  $x = 250$ ; So, they will meet at a distance of 250 km from place A
- So the time at which they will meet will be  $(250/50) = 5$  hrs (after 6 am)  
Hence, they will meet at 11 am.

**Question 20:** A train 125 m long passes a man, running at 5 km/hr in the same direction in which the train is going, in 10 seconds. The speed of the train is:



- a. 49 km/hr                      b. 50 km/hr
- c. 51 km/hr                      d. 52 km/hr

**Explanation:**

- Speed of the train relative to man =  $(125/10)$  m/sec =  $(25/2)$  m/sec.  $[(25/2) * (18/5)]$   
km/hr = 45 km/hr.
- Let the speed of the train be  $x$  km/hr.
- Then, relative speed =  $(x - 5)$  km/hr.  $x - 5 = 45 \implies x = 50$  km/hr.

**Question 21:** A train runs with a speed of 200 km/hour. What will be its speed in m/sec?

- a. 50 m/sec                      b. 55.55 m/sec
- c. 48 m/sec                      d. 66.66 m/sec

**Explanation:**

- Speed in km/ hour =200.
- Speed in m/sec =  $200 \times \frac{5}{18}$ ;  $500/9$  m/sec = 55.55 m/sec

**Question 22:** How long does a train 110 meters long running at the speed of 72 km/hour take to cross a bridge 132 meters in length?

- a. 15 seconds                      b. 12.1 seconds
- c. 10 seconds                     d. 8.1 seconds

**Explanation:**

- Speed = 72 km/hour =  $72 \times (5/18)$  m/sec = 20 m/sec
- Total distance to be covered = 110 + 132 = 142 meters
- Time = Distance/Speed =  $142/20 = 7.1$  seconds

**Question 23:** A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a telegraph post, then the length of the train is:

- a. 90 m                      b. 120 m

b. 100 m

d. 140 m

**Explanation:**

→ Speed =  $12/10 \times 60 \text{ km/hr} = 72 \times 5/18 \text{ m/sec} = 20 \text{ m/sec}$ .

→ Length of the train = (Speed x Time) =  $(20 \times 6) \text{ m} = 120 \text{ m}$ .

**Question 24:** A train having a length of 240 meter passes a post in 24 seconds. How long will it take to pass a platform having a length of 650 meters?

a. 120 seconds

b. 99 seconds

c. 89 seconds

d. 80 seconds

**Explanation:**

→ Speed of the train =  $240/24 = 10 \text{ m/s}$

→ Required time =  $240 + 650/10 = 89 \text{ seconds}$

**Question 25:** A train 240 m long passes a pole in 24 seconds. How long will it take to pass a platform 650 m long?

a. 65 sec

b. 89 sec

c. 100 sec

c. 100 sec

**Explanation:**

→ Speed =  $240/24 \text{ m/sec} = 10 \text{ m/sec}$ .

→ Required time =  $(240 + 650)/10 \text{ sec} = 89 \text{ sec}$ .

## 7.6 ASSIGNMENT - PROBLEMS ON TRAINS

1. How long does a train 100 meters long running at the rate of 40 km/hr take to cross a telegraphic pole?

- a. 9 hrs
- b. 9 min
- c. 9 sec
- d. 9.9 hrs

2. There is a big square shaped haunted palace in a jungle. A train 1000 meters long passes it with a speed of 120 km/hr in a minute. Find the area of the haunted palace.

- a. 1 sq.km
- b. 800 sq. m.
- c. 1.5 sq.km
- d. 1.2 sq.km

3. A car travels from P to Q with 15 km/hr and returns with 30 km/hr. What is the average speed of journey?

- a. 22.5 km/hr
- b. 20 km/hr
- c. 25 km/hr
- d. 10 km/hr

4. Three persons decided to go on a long tour and decided to drive the same distance of 100 km each. First person drove at the speed of 60 km/hr. After a break, the second person drove with the speed increased by 10 km/hr. Then at the end, the third person drove again with the speed increased by 10 km/hr. What is the average approximate speed of their journey?

- a. 75 km/hr
- b. 69 km/hr
- c. 100 km/hr
- d. none of these

5. A cycled from P to Q at 10 km/hr & returned at the rate of 9 km/hr. B cycled both ways at 12 km/hr in the whole journey. B took 10min less than A. find the distance between P & Q.

- a. 1.50 km
- b. 3.75 km
- c. 4.50 km
- d. 4.98 km

**6.** A person during his journey travels 40 minutes at a speed of 30 km/hr, another 50 minutes at 60 km/hr and last 1 hr at 30 km/hr. What is his average speed?

- a. 38 km/hr
- b. 40 km/hr
- c. 60 km/hr
- d. 35.7 km/hr

**7.** Sohan left his house at 7 am and walked at the rate of 2 km/hr to his college. But he saw that his college was closed and immediately he returned at the rate of 3 km/hr. If he had reached his house at 10 am, what is the distance of his college from his house?

- a. 3 km
- b. 5 km
- c. 4 km
- d. 3.6 km

**8.** By walking at  $\frac{3}{4}$ th of his usual speed Hemant is 6 minutes late to his office. What is his usual time to cover the distance?

- a. 20 min
- b. 18 min
- c. 24 min
- d. 1 hr

**9.** A train 145 meters long is running with a speed of 25 km/hr. In what time will it pass a man who is walking at 4 km/hr in the direction opposite to that of the train?

- a. 10 sec
- b. 8 sec
- c. 9 sec
- d. 18 sec

**10.** Two trains of lengths 100 meters and 120 meters are running in the same direction with speeds 30 km/hr and 40 km/hr respectively. In what time (approx.) will they pass each other?

- a. 24 sec
- b. 79 sec
- c. 80 sec
- d. 144 sec

**11.** Two persons are running in the same direction with the speeds of 3 km/hr and 6 km/hr. A truck comes running from behind and passes them in 9 and 10 seconds respectively. What is the speed of the truck?

- a. 40 km/hr
- b. 22 km/hr
- c. 11 km/hr
- d. 33 km/hr

**12.** A circular playground has an area 616 sq. meters. What time will it take for a runner to run around the circular ground at a speed of 22 km/hr?

- a. 14 hrs
- b. 14.4 sec
- c. 16.2 sec
- d. none of these

**13.** A train goes from P to Q in an hour and another train comes from Q to P in 1.5 hours. After what time will they meet each other?

- a. 30 min
- b. 36 min
- c. 42 min
- d. 48 min

**14.** Two trains starting at the same time from two stations 200 km apart, and going in opposite directions cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?

- a. 9:20
- b. 11:20
- c. 11:9
- d. 1:2

**15.** A cat can climb 6 meters of a round pole in one minute, but slips off 3 meters in the next minute. What time will it take to reach the top of a pole which is 21 meters long?

- a. 14 min
- b. 12 min
- c. 11 min
- d. none of these

**16.** A thief steals a car at 2.30 p.m. and drives it at 60 km/hr. The theft is discovered at 3 p.m. and the owner sets off in another car at 75 km/hr. When will he overtake the thief?

- a. 4.30 p.m.
- b. 4.45 p.m.
- c. 5 p.m.
- d. 5.15 p.m.

**17.** A and B start from house at 10am. They travel from the same point at 20 km/h and 40 km/h. There is a Junction T on their path. A turns left at T junction at 12:00noon. B reaches T earlier, and turns right. Both of them continue to travel till 2pm. What is the distance between A and B at 2.00 pm?

- a. 160 Km
- b. 120 Km
- c. 80 Km
- d. 100 Km

**18.** Two cars travel simultaneously from the cities P and Q towards each other. Their speeds are 20 m/s and 25 m/s respectively. If  $d(P, Q) = 900$  km, find at what distance from Q will they meet?

- a. 400 km
- b. 500 km
- c. 250 km
- d. 200 km

**19.** Two trains travel toward each other on the same track, beginning 100 miles apart. One train travels at 40 miles per hour; the other travels at 60 miles an hour. A bird starts flight at the same location as the faster train, flying at a speed of 90 miles per hour. When it reaches the slower train, it turns around, flying the other direction at the same speed. When it reaches the faster train again, it turns around and so on. When the trains collide, how far will the bird have flown?

- a. 50 miles
- b. 72 miles
- c. 118 miles
- d. 90 miles

**20.** A nonstop bus to Mumbai overtakes an auto also moving towards Mumbai at 10 am. The bus reaches Mumbai at 12.30 pm and starts on the return journey after 1 hr. On the way back it meets the auto at 2 pm. At what time the auto will reach Mumbai?

a. 2.30 pm

b. 3 pm

c. 3.30 pm

d. cannot be determined

**21.** The speed of a swimmer in still water is 5 km/hr and the speed of the river is 1 km/hr. Find the upstream and downstream speeds of the swimmer.

a. (5, 1) km/hr

b. (4, 6) km/hr

c. (6, 4) km/hr

d. none of these

**22.** A boat is rowed down a river 40 km in 5 hours and up the river 21 km in 7 hours. What is the speed of the boat and the river?

a. (5.5, 2.5) km/hr

b. (5, 7) km/hr

c. (8, 3) km/hr

d. (11, 5) km/hr

**23.** A man rows at a speed of 8 km/hr in still water to a certain distance upstream and back to the starting point in a river flowing at 4 km/hr. What is the average speed of the journey?

a. 6 km/hr

b. 4 km/hr

c. 32 km/hr

d. 8 km/hr

**24.** A man can row 7 km/hr in still water. If the river is flowing at 3 km/hr, it takes 6 hours more in upstream than to go downstream for the same distance. How far is the place?

a. 21 km

b. 30 km

c. 40 km

d. 10 km

**25.** A man can row 6 km/hr in still water. If the river is flowing with 2 km/hr, it takes him 3 hours to row to a place and back. How far is the place?

a. 8 km

b. 12 km

c. 11 km

d. 5 km

**7.6.1 Answer Keys- Problems on Trains**

<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>
<b>1</b>	c	<b>6</b>	b	<b>11</b>	d	<b>16</b>	c	<b>21</b>	b
<b>2</b>	a	<b>7</b>	d	<b>12</b>	b	<b>17</b>	a	<b>22</b>	a
<b>3</b>	b	<b>8</b>	b	<b>13</b>	b	<b>18</b>	b	<b>23</b>	a
<b>4</b>	b	<b>9</b>	d	<b>14</b>	c	<b>19</b>	d	<b>24</b>	c
<b>5</b>	b	<b>10</b>	b	<b>15</b>	c	<b>20</b>	b	<b>25</b>	a





## **UNIT - 8**

### **PERMUTATION, COMBINATION AND PROBABILITY**

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#### **Learning Objective**

**After going through this unit, you will be able to understand:**

- What is the basic idea of Permutation, Combination and Probability
  - Moderate questions on Permutation, Combination and Probability
  - Explain what is factorial and its application
  - To define event and calculating the probability that an event will occur
- 

#### **8.1 Theoretical Framework of Permutation and Combination**

##### **What is Permutation?**

Permutation relates to the act of arranging all the members of a set into some sequence or order, or if the set is already ordered, rearranging its elements, a process called permuting. Permutations occur, in more or less prominent ways, in almost every area of mathematics. They often arise when different orderings on certain finite sets are considered.

##### **What is Combination?**

Combination is a way of selecting items from a collection, such that (unlike permutations) the order of selection does not matter. In smaller cases, it is possible to count the number of combinations. Combination refers to the combination of  $n$  things taken  $k$  at a time without repetition.

## Formulas

There are many formulas involved in permutation and combination concept. The two key formulas are:

### Permutation Formula

A permutation is the choice of  $r$  things from a set of  $n$  things without replacement and where the order matters.

$${}_nP_r = (n!) / (n-r)!$$

### Combination Formula

A combination is the choice of  $r$  things from a set of  $n$  things without replacement and where order doesn't matter.

$${}_nC_r = \binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

#### 1. Factorial Notation:

Let  $n$  be a positive integer. Then, factorial  $n$ , denoted  $n!$  is defined as:

$$n! = n(n-1)(n-2) \dots 3.2.1.$$

#### Examples:

1. We define  $0! = 1$ .
2.  $4! = (4 \times 3 \times 2 \times 1) = 24$ .
3.  $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$ .

## 8.2: SOLVED EXAMPLES - PERMUTATIONS AND COMBINATIONS

**Question 1:** A college has 10 basketball players. A 5-member team and a captain will be selected out of these 10 players. How many different selections can be made?

- a. 1260
- b. 210
- c.  ${}^{10}C_6 \times 6!$
- d.  ${}^{10}C_5 \times 6$

**Explanation:**

- A team of 6 members has to be selected from the 10 players. This can be done in  ${}^{10}C_6$  or 210 ways.
- Now, the captain can be selected from these 6 players in 6 ways.
- Therefore, total ways the selection can be made is  $210 \times 6 = 1260$ .

**Question 2:** Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

- a. 24400
- b. 21300
- c. 210
- d. 25200

**Explanation:**

- Number of ways of selecting 3 consonants from 7 =  ${}^7C_3$
- Number of ways of selecting 2 vowels from 4 =  ${}^4C_2$
- Number of ways of selecting 3 consonants from 7 and 2 vowels from 4
- $= {}^7C_3 \times {}^4C_2$
- $= (7 \times 6 \times 5 / 3 \times 2 \times 1) \times (4 \times 3 / 2 \times 1) = 210$
- It means we can have 210 groups where each group contains a total of 5 letters (3 consonants and 2 vowels).
- Number of ways of arranging 5 letters among themselves =  $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- Hence, required number of ways =  $210 \times 120 = 25200$

**Question 3:** A boy has nine trousers and 12 shirts. In how many different ways can he select a trouser and a shirt?

- a. 21
- b. 12

c. 9

d. 108

**Explanation:**

- The boy can select one trouser in nine ways.
- The boy can select one shirt in 12 ways.
- The number of ways in which he can select one trouser and one shirt is  $9 \times 12 = 108$  ways.

**Question 4. Find the sum of all the 4 digit numbers that can be formed with the digits 3, 4, 5 and 6**

a. 119988

b. 11988

c. 191988

d. None of these

**Explanation:**

- No. of Digits = 4 All are distinct; They can be arranged in  $4! = 24$  ways
- Each of the digits 3, 4, 5 and 6 occur at unit place =  $3!$  Ways = 6 ways.
- Thus there will be 6 numbers ending with 3, 4, 5 and 6 each. So the sum of the digits at unit's place =  $6(3 + 4 + 5 + 6) = 108$
- The sum of numbers =  $108 \times 103 + 108 \times 102 + 108 \times 101 + 108 \times 100 = 119988$

**5. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?**

a. 720

b. 520

c. 700

d. 750

**Explanation:**

- The word 'LEADING' has 7 different letters.
- When the vowels EAI are always together, they can be supposed to form one letter.
- Then, we have to arrange the letters LNDG (EAI).
- Now, 5 (4 + 1) letters can be arranged in  $5! = 120$  ways.
- The vowels (EAI) can be arranged among themselves in  $3! = 6$  ways.
- Required number of ways =  $(120 \times 6) = 720$ .

**6. How many combinations of students are possible if the group is to consist of exactly 3 freshmen?**

- a. 5000  
b. 4550  
c. 4000  
d. 3550

**Explanation:**

- Here we need the number of possible combinations of 3 out of 5 freshmen,  ${}^5C_3$ , and the number of possible combinations of 3 out of the 15 sophomores and juniors,  ${}^{15}C_3$ .
- Note that we want 3 freshmen and 3 students from the other classes. Therefore, we multiply the number of possible groups of 3 of the 5 freshmen times the number of possible groups of 3 of the 15 students from the other classes.  ${}^5C_3 \times {}^{15}C_3 = 4,550$ .

7. Find the number of words, with or without meaning, that can be formed with the letters of the word 'SWIMMING'?

- a. 10080                      b. 4569  
c. 9800                        d. 2478

**Explanation:**

- The word 'SWIMMING' contains 8 letters. Of which, I occur twice and M occurs twice.
- Therefore, the number of words formed by this word =  $8! / (2! * 2!) = 10080$ .

8. A five-digit number is formed using digits 1, 3, 5, 7 and 9 without repeating any one of them. What is the sum of all such possible numbers?

- a. 6666600                      b. 6666660  
c. 6666666                      d. None of these

**Explanation:**

- If we fix 1 at the unit's place, the other digits can be arranged in  $4! = 24$  ways.
- So, there are 24 numbers which have 1 at the unit's place and the sum of these numbers is 24.
- Similarly, there will be 24 numbers each with 3, 5, 7 and 9 at the unit's place.
- So, the sum of all the numbers at the unit's place will be:  
 $24 + 24 \times 3 + 24 \times 5 + 24 \times 7 + 24 \times 9 = 24 + 72 + 120 + 168 + 216 = 600$ .
- Similarly, the sum of the digits at the ten's place, hundreds place,  
thousand's place and ten thousand's place will be 600.

→ So, the sum of all numbers =  $600 (1 + 10 + 100 + 1,000 + 10,000) = 6666600$ .

**9. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first digit of the code is nonzero. The code, handwritten on a slip, can however potentially create confusion, when read upside down for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?**

- |       |       |
|-------|-------|
| a. 80 | b. 78 |
| c. 71 | d. 69 |

**Explanation:**

- The available digits are 0, 1, 2, ..., 9.
- The first digit can be chosen in 9 ways (0 not acceptable), the second digit can be accepted in 9 ways (digits repetition not allowed)
- Thus, the code can be made in  $9 \times 9 = 81$  ways.
- Now there are only 4 digits 1, 6, 8, 9 which can create confusion.
- Hence, the total number of codes which create confusion are  $= 4 \times 3 = 12$ .
- Out of these 12 codes 69 and 96 will not create confusion.
- Hence, in total  $12 - 2 = 10$  codes will create confusion.
- Hence, the total codes without confusion are  $81 - 10 = 71$ .

**10. A college has 10 basketball players. A 5-member team and a captain will be selected out of these 10 players. How many different selections can be made?**

- |         |         |
|---------|---------|
| a. 1260 | b. 1400 |
| c. 1250 | d. 1600 |

**Explanation:**

- A team of 6 members has to be selected from the 10 players. This can be done in  ${}^{10}C_6$  or 210 ways.
- Now, the captain can be selected from these 6 players in 6 ways.
- Therefore, total ways the selection can be made is  $210 \times 6 = 1260$ .

**11. How many words can be formed by using all letters of TIHAR?**

- |        |        |
|--------|--------|
| a. 100 | b. 120 |
| c. 140 | d. 160 |

**Explanation:**

- First thing to understand in this question is that it is a permutation question.
- Total number of words = 5
- Required number =  $5P5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

**12. How many 3-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?**

- a. 720
- b. 420
- c. None of these
- d. 5040

**Explanation:**

- The word 'LOGARITHMS' has 10 different letters.
- Hence, the number of 3-letter words (with or without meaning) formed by using these letters
- $= {}^{10}P_3$
- $= 10 \times 9 \times 8 = 720$

**13. In how many different ways can the letters of the word 'DETAIL' be arranged such that the vowels must occupy only the odd positions?**

- a. None of these
- b. 64
- c. 120
- d. 36

**Explanation:**

- The word 'DETAIL' has 6 letters which has 3 vowels (EAI) and 3 consonants(DTL)
- The 3 vowels(EAI) must occupy only the odd positions. Let's mark the positions as (1) (2) (3) (4) (5) (6). Now, the 3 vowels should only occupy the 3 positions marked as (1),(3) and (5) in any order.
- Hence, number of ways to arrange these vowels
- $= {}^3P_3 = 3! = 3 \times 2 \times 1 = 6$
- Now we have 3 consonants(DTL) which can be arranged in the remaining 3 positions in any order. Hence, number of ways to arrange these consonants
- $= {}^3P_3 = 3! = 3 \times 2 \times 1 = 6$
- Total number of ways



→  $= 6 \times 6 = 36$

a. 810                      b. 1440  
c. 2880                     d. 50400

- In the word 'CORPORATION', we treat the vowels OOAIO as one letter.
- Thus, we have CRPRTN (OOAIO).
- This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different.
- Number of ways arranging these letters =  $7! = 2520 \cdot 2!$
- Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged
- in  $5! / 3! = 20$  ways.
- Required number of ways =  $(2520 \times 20) = 50400$ .

a. 119988                      b. 11988

c. 191988                     d. None of these

- No. of Digits = 4 All are distinct; They can be arranged in  $4! = 24$  ways
- Each of the digits 3, 4, 5 and 6 occur at unit place =  $3!$  Ways = 6 ways.
- Thus there will be 6 numbers ending with 3, 4, 5 and 6 each. So the sum of the digits at unit's place =  $6(3 + 4 + 5 + 6) = 108$
- The sum of numbers =  $108 \times 103 + 108 \times 102 + 108 \times 101 + 108 \times 100 = 119988$

a. 7                                      b. 9  
c. 10                                     d. None of these

→ Let  $n$  be the number of teams.

$$\rightarrow {}^nC_2 = 21$$

$$\rightarrow (n(n-1)/2) = 21$$

$$\rightarrow n(n-1) = 42; n = 7$$

**17. A committee has 5 men and 6 women. What are the number of ways of selecting 2 men and 3 women from the given committee?**

a. 150

b. 200

c. 250

d. 300

**Explanation:**

$$\rightarrow \text{The number of ways to select two men and three women} = {}^5C_2 * {}^6C_3$$

$$\rightarrow = (5 * 4) / (2 * 1) * (6 * 5 * 4) / (3 * 2)$$

$$\rightarrow = 200$$

**18. There are 18 stations between Hyderabad and Bangalore. How many second class tickets have to be printed, so that a passenger can travel from any station to any other station?**

a. 200

b. 190

c. 95

d. 100

e. 380

**Explanation:**

$$\rightarrow \text{The total number of stations} = 20$$

$\rightarrow$  From 20 stations we have to choose any two stations and the direction of travel (i.e., Hyderabad to Bangalore is different from Bangalore to Hyderabad). in  ${}^{20}P_2$  ways.

$$\rightarrow {}^{20}P_2 = 20 * 19 = 380.$$

**19. When four fair dice are rolled simultaneously, in how many outcomes will at least one of the dice show 3?**

a. 155

b. 620

c. 671

d. 625

**Explanation:**

$\rightarrow$  When 4 dice are rolled simultaneously, there will be a total of  $6^4 = 1296$  outcomes.

$\rightarrow$  The number of outcomes in which none of the 4 dice show 3 will be  $5^4 = 625$  outcomes.

$\rightarrow$  Therefore, the number of outcomes in which at least one die will show 3 =  $1296 - 625 = 671$

**20. Four dice are rolled simultaneously. What is the number of possible outcomes in which at least one of the die shows 6?**

- a.  $6! / 4!$
- b. 625
- c. 671
- d. 1296

**Explanation:**

- When 4 dice are rolled simultaneously, there are  $6^4 = 1296$  outcomes.
- The converse of what is asked in the question is that none of the dice show '6'.
- That is all four dice show any of the other 5 numbers.
- That is possible in  $5^4 = 625$  outcomes.
- Therefore, in  $1296 - 625 = 671$  outcomes at least one of the dice will show 6.

**21. Find the number of ways in which one or more letters be selected from the letters AAAABBCCCEDEF.**

- a. 479
- b. 480
- c. 958
- d. 960

**Explanation:**

- The number of ways of
- Selecting A =  $4+1 = 5$
- Selecting B =  $2+1 = 3$
- Selecting C =  $3+1 = 4$
- Selecting D =  $1+1 = 2$
- Selecting E =  $1+1 = 2$
- Selecting F =  $1+1 = 2$
- Hence, total ways of selecting =  $5 \times 3 \times 4 \times 2 \times 2 \times 2 - 1 = 480 - 1 = 479$

**22. How many different four letter words can be formed (the words need not be meaningful) using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R?**

- a. 52
- b. 56

c. 59

d. none of these

***Explanation:***

- The first letter is E and the last one is R.
- Therefore, one has to find two more letters from the remaining 11 letters.
- Of the 11 letters, there are 2 Ns, 2Es and 2As and one each of the remaining 5 letters.
- The second and third positions can either have two different letters or have both the letters to be the same.
- Case 1: When the two letters are different. One has to choose two different letters from the 8 available different choices. This can be done in  $8 * 7 = 56$  ways.
- Case 2: When the two letters are the same. There are 3 options - the three can be either Ns or Es or As. Therefore, 3 ways.
- Total number of possibilities =  $56 + 3 = 59$ .

***23. Find the number of different words that can be formed with the letters of the word 'BUTTER' so that the vowels are always together.***

- a. 120 ways
- b. 150 ways
- c. 80 ways
- d. 25 ways

***Explanation:***

- The word 'BUTTER' contains 6 letters.
- The letters U and E should always come together. So the letters are B, T, T, R, (UE).
- Number of ways in which the letters above can be arranged =  $5!/2! = 60$  (since the letter 'T' is repeated twice).
- Number of ways in which U and E can be arranged =  $2! = 2$  ways
- Therefore, total number of permutations possible =  $60 \times 2 = 120$  ways.

***24. The Indian Cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket eleven be selected if we have to select 1 wicket keeper and at least 4 bowlers?***

- a. 1024
- b. 1900
- c. 2000
- d. 1092

***Explanation:***

- We are to choose 11 players including 1 wicket keeper and 4 bowlers or, 1 wicket keeper and 5 bowlers.
- Number of ways of selecting 1 wicket keeper, 4 bowlers and 6 other players in  ${}^2C_1 \times {}^5C_4 \times {}^9C_6 = 840$
- Number of ways of selecting 1 wicket keeper, 5 bowlers and 5 other players in  ${}^2C_1 * {}^5C_5 * {}^9C_5 = 252$
- Total number of ways of selecting the team =  $840 + 252 = 1092$

**25. In how many words can be formed by using all letters of the word BHOPAL**

- |        |        |
|--------|--------|
| a. 420 | b. 520 |
| c. 620 | d. 720 |

***Explanation:***

- The word 'BHOPAL' contains 6 letters.
- Number of ways in which the letters above can be arranged =  $6!$
- Therefore, total number of permutations possible =  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$  ways.

**8.** How many words, with or without meaning, can be formed out of the letters of the word, CORRESPONDENCE?

a.  $14! / 2! \times 2! \times 2! \times 3!$

b.  $14! / 2! \times 2! \times 3!$

c.  $14! / 2! \times 3! \times 3!$

d.  $14! / 2! \times 2! \times 2!$

**9.** There are 3 red, 2 yellow and 2 green flags. Using all the flags at a time, how many signals can be transmitted?

a. 7

b. 12

c. 35

d. 210

**10.** How many 9 digit numbers are possible by using the digits 1, 2, 3, 4, 5 which are divisible by 4 if the repetition of the digits is allowed?

a.  $5^7$

b.  $5^6$

c.  $5^9$

d.  $5^8$

**11.** There are three boys and three girls. The boy *A* does not want any girl neighbor and the girl *B* does not want any boy neighbor. In how many ways can they be arranged round a table?

a. 1

b. 2

c. 3

d. 4

**12.** There are 5 gentlemen and 4 ladies standing in a circle. If no two ladies can stand next to each other, in how many ways can they stand?

a. 2000

b. 2880

c. 3000

d. 3400

**13.** In an examination, there are two groups of questions containing 4 questions each. A student has to attempt 5 questions in all, but he cannot attempt more than 3 questions from any group. In how many ways can he attempt the questions?

a. 28

b. 45

c. 48

d. 60

**14.** There are 21 points on a circle. How many straight lines can be drawn from them?

a. 20

b. 168

c. 210

d. 288

**15.** There are 10 points on a straight line AB and 8 points on a straight line AC, none of them being A, B and c. How many triangles can be formed using these points as vertices?

a. 504

b. 228

c. 640

d. 748

**16.** How many rectangles can be formed by the intersection of 7 parallel horizontal lines and 6 parallel vertical lines?

a. 235

b. 315

c. 427

d. 502

**17.** Find the total number of rectangles on a chess board.



- a. 64  
c. 888
- b. 128  
d. 1296

**18.** In a boxing championship, 153 matches were played in total. If every boxer plays a match with every other boxer, how many boxers have participated in the championship?

- a. 17  
c. 19
- b. 18  
d. 20

**19.** A grandfather has 5 sons and daughters and 8 grandchildren. They have to be arranged in a row such that the first 4 seats and the last 4 seats are to be taken by grandchildren and the grandfather would not sit adjacent to any of the grandchildren. In how many ways can this be done?

- a.  $8! \times 4! \times 5!$   
c.  $8! \times 5! \times 4$
- b.  $8 \times 5! \times 4!$   
d.  $8! \times 5 \times 4!$

**20.** There are 4 married couples out of which 3 a group is needed. But there should not be his or her spouse. How many groups are possible?

- a. 12  
c. 32
- b. 22  
d. 42

### 8.3.1 Answer Keys - Permutation and Combinations

Q. No	Ans	Q. No	Ans	Q. No	Ans	Q. No	Ans
1	a	6	b	11	d	16	b
2	b	7	c	12	b	17	d
3	c	8	a	13	c	18	b
4	b	9	d	14	c	19	c
5	d	10	d	15	c	20	c

## 8.4 PROBABILITY

Probability means possibility, it deals with the occurrence of a random event. The value is expressed between zero and one. Probability has been introduced in Maths to predict how likely events are to happen.

The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment. To find the probability of a single event to occur, first we should know the total number of possible outcomes.

For example, when we toss a coin, either we get Head OR Tail, only two possible outcomes are possible (H, T). But if we toss two coins in the air, there could be three possibilities of events to occur, such as both the coins show heads or both show tails or one shows heads and one tail, i.e.(H,H), (H,T),(T,T).

### Formula for Probability

The probability formula is defined as the possibility of an event to happen is equal to the ratio of the number of favorable outcomes and the total number of outcomes.

Probability of event to happen  $P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total Number of outcomes}}$

---

Sometimes students get mistaken for “favorable outcome” with “desirable outcome”. This is the basic formula. But there are some more formulas for different situations or events.

## 8.5: SOLVED EXAMPLES- PROBABILITY

**Question 1:** Sam and Joan are playing a tennis match. If the probability of Sam's win is 0.59, then find the probability of Joan's win.

- a. 0.47
- b. 0.36
- c. 0.41
- d. 0.25

**Explanation:**

- Let event A = Sam wins and event B = Joan wins.
- Then,  $P(A) = 0.59$
- Since if Sam wins, Joan cannot win and if Joan wins, Sam cannot win, so we can say that the events A and B are mutually exclusive. Other than these two events, there are no any other possible outcomes.
- So,  $P(A) + P(B) = 1$   
 $0.59 + P(B) = 1$   
 $P(B) = 1 - 0.59 = 0.41$

**Question 2:** Let A and B be events on the same sample space, with  $P(A) = 0.6$  and  $P(B) = 0.7$ . Can these two events be disjoint?

- a. Yes
- b. No
- c. None of the above
- d. All the above

**Explanation:**

- These two events cannot be disjoint because  $P(A) + P(B) > 1$ .
- $P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$ .
- An event is disjoint if  $P(A \text{ and } B) = 0$ . If A and B are disjoint  $P(A \text{ or } B) = 0.6 + 0.7 = 1.3$   
And Since probability cannot be greater than 1, these two mentioned events cannot be disjoint.

**Question 3:** A family has two children. find the probability that both children are girls given that at least one of them is a girl?

- a.  $1/4$
- b.  $2/3$
- c.  $1/3$
- d.  $1/4$

**Explanation:**

Let b stand for boy and g for girl.

The sample space of the experiment  $S = \{(g, g), (g, b), (b, g), (b, b)\}$

Let E and F denote the following events :

E : ‘both the children are girls’

F : ‘at least one of the child is a girl’

Then  $E = \{(g, g)\}$  and  $F = \{(g, g), (g, b), (b, g)\}$

Now  $E \cap F = \{(g, g)\}$

Thus  $P(F) = 3/4$

and  $P(E \cap F) = 1/4$

Therefore  $P(E|F) = P(E \cap F)/P(F) = (1/4)/(3/4) = 1/3$

**Question 4:** In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?

- a.  $1/10$
- b.  $2/5$
- c.  $2/7$
- d.  $5/7$

**Explanation:**

$P(\text{getting a prize}) = 10 / (10+25) ; 10 / 35 = 2 / 7.$

**Question 5:** Determine the probability that a digit chosen at random from the digits 1, 2, 3, ...12 will be odd.

- a.  $1/2$
- b.  $1/9$
- c.  $5/9$
- d.  $4/9$

**Explanation:**

- Total no. of Digits = 12. Equally likely cases = 12.
- There are six odd digits. Probability =  $6 / 12 = 1 / 2$

**Question 6:** Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

- a.  $1/2$
- b.  $2/5$
- c.  $8/15$
- d.  $9/20$

**Explanation:**

- Here,  $S = \{1, 2, 3, 4, \dots, 19, 20\}$ .
- Let E = event of getting a multiple of 3 or 5 =  $\{3, 6, 9, 12, 15, 18, 5, 10, 20\}$ .
- $P(E) = n(E)/n(S) = 9/20$ .

**Question 7:** Which of these numbers cannot be a probability?

- a. -0.00001
- b. 0.5
- c. 1.001
- d. 1

**Explanation:**

- A probability is always greater than or equal to 0 and less than or equal to 1, hence only a. and c. above cannot represent probabilities: -0.00010 is less than 0 and 1.001 is greater than 1.

**Question 8:** A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, NOT replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen?

- a.  $2/14$
- b.  $1/14$
- c.  $6/17$
- d.  $8/12$

***Explanation:***

- Probability of drawing 1 blue pen =  $\frac{4}{9}$
- Probability of drawing another blue pen =  $\frac{3}{8}$
- Probability of drawing 1 black pen =  $\frac{3}{7}$
- Probability of drawing 2 blue pens and 1 black pen =  $\frac{4}{9} * \frac{3}{8} * \frac{3}{7} = \frac{1}{14}$

***Question 9: In a throw of dice what is the probability of getting a number greater than 5***

- a.  $\frac{1}{2}$
- b.  $\frac{1}{3}$
- c.  $\frac{1}{5}$
- d.  $\frac{1}{6}$

***Explanation:***

- Number greater than 5 is 6, so only 1 number
- Total cases of dice = [1,2,3,4,5,6]
- So probability =  $\frac{1}{6}$

***Question 10: The probability that Soumya will get marry within 365 days is 'a' and the probability that her colleague Alma get marry within 365 days is 'b'. Find the probability that only one of the two gets married at the end of 365 days.***

- a.  $a-b-2ab$
- b.  $a+b-2ab$
- c.  $a-b+2ab$
- d.  $ab-a-b$

***Explanation:***

- The probability that Soumya will get marry than Alma will be =  $a(1-b)$ .. Similarly, the probability that Alma will get marry than Soumya will be  $b(1-a)$ . The probability that either of these two get marry =  $a(1-b) + b(1-a) = a+b-2ab$ .

***Question 11: Three unbiased coins are tossed. What is the probability of getting at most two heads?***

- a.  $\frac{3}{4}$
- b.  $\frac{1}{4}$

c.  $3/8$

d.  $7/8$

**Explanation:**

→ Here  $S = \{TTT, TTH, THT, HTT, THH, HTH, HHT, HHH\}$

→ Let  $E$  = event of getting at most two heads.

→ Then  $E = \{TTT, TTH, THT, HTT, THH, HTH, HHT\}$ .

→  $P(E) = n(E)/n(S) = 7/8$ .

**Question 12:** Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random.

What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

a.  $1/2$

b.  $3/5$

c.  $9/20$

d.  $8/15$

**Explanation:**

→ Here,  $S = \{1, 2, 3, 4, \dots, 19, 20\}$ .

→ Let  $E$  = event of getting a multiple of 3 or 5 =  $\{3, 6, 9, 12, 15, 18, 5, 10, 20\}$ .

→  $P(E) = n(E)/n(S) = 9/20$ .

**Question 13:** In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random.

What is the probability of getting a prize?

a.  $2/7$

b.  $5/7$

c.  $1/5$

d.  $1/2$

**Explanation:**

→ Total number of outcomes possible,  $n(S) = 10 + 25 = 35$

→ Total number of prizes,  $n(E) = 10$

→  $P(E) = n(E)/n(S) = 10/35 = 2/7$

**Question 14:** A bag contains 12 white and 18 black balls. Two balls are drawn in succession without replacement. What is the probability that the first is white and the second is black?

a.  $18/145$

b.  $18/29$

c.  $36/135$

d.  $36/145$

***Explanation:***

→ The probability that first ball is white:

$$= \frac{{}^{12}C_1}{{}^{30}C_1}$$

$$= \frac{12}{30}$$

$$= \frac{2}{5}$$

→ Since the ball is not replaced; hence the number of balls left in bag is 29.

Hence, the probability the second ball is black:

$$= \frac{{}^{18}C_1}{{}^{29}C_1}$$

$$= \frac{18}{29}$$

→ Required probability,

$$= \left(\frac{2}{5}\right) \times \left(\frac{18}{29}\right)$$

$$= \frac{36}{145}$$

***Question 15: A bag contains 5 red and 3 green balls. Another bag contains 4 red and 6 green balls. If one ball is drawn from each bag. Find the probability that one ball is red and one is green.***

a.  $19/20$

b.  $17/20$

c.  $8/10$

d.  $21/40$

***Explanation:***

Let A be the event that the ball selected from the first bag is red and the ball selected from the second bag is green.

Let B be the event that the ball selected from the first bag is green and the ball selected from the second bag is red.

$$P(A) = \left(\frac{5}{8}\right) \times \left(\frac{6}{10}\right) = \frac{3}{8} \text{ and } P(B) = \left(\frac{3}{8}\right) \times \left(\frac{4}{10}\right) = \frac{3}{20}$$

Hence, required probability,

$$= P(A) + P(B)$$

$$= \frac{3}{8} + \frac{3}{20}$$

$$= \frac{21}{40}$$



**Question 16:** *If a number is chosen at random from 1 to 100, then the probability that the chosen number is a perfect cube is:*

- a.  $4/13$
- b.  $1/25$
- c.  $1/2$
- d.  $1/10$

**Explanation:**

- We have 1, 8, 27 and 64 as perfect cubes from 1 to 100.
- Thus, the probability of picking a perfect cube is  $4/100 = 1/25$

**Question 17:** *If a number is chosen at random from 1 to 100, then the probability that the chosen number is a perfect cube is:*

- a.  $4/13$
- b.  $1/25$
- c.  $1/2$
- d.  $1/10$

**Explanation:**

- We have 1, 8, 27 and 64 as perfect cubes from 1 to 100.
- Thus, the probability of picking a perfect cube is  $4/100 = 1/25$

**Question 18:** *What is the probability of getting a sum 9 from two throws of dice?*

- a.  $1/3$
- b.  $1/9$
- c.  $1/12$
- d.  $2/9$

**Explanation:**

- Total number of cases =  $6 \times 6 = 36$
- Favoured cases =  $[(3,6), (4,5), (6,3), (5,4)] = 4$
- So probability =  $4/36 = 1/9$

**Question 19:** A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is:

- a.  $1/13$
- b.  $2/13$
- c.  $1/26$
- d.  $1/52$

**Explanation:**

- Total number of cases = 52
- Favorable cases = 2
- Probability =  $2/52 = 1/26$

**Question 20:** A coin is tossed five times. What is the probability that there is at least one tail?

- a.  $31/32$
- b.  $1/16$
- c.  $1/2$
- d.  $1/32$

**Explanation:**

- Let  $P(T)$  be the probability of getting at least one tail when the coin is tossed five times.
- There is not even a single tail.
- i.e. all the outcomes are heads. =  $1/32$  ;  $P(T) = 1 - 1/32 = 31/32$

**Question 21:** Consider another example where a pack contains 4 blue, 2 red and 3 black pens. If a pen is drawn at random from the pack, replaced and the process repeated 2 more times, What is the probability of drawing 2 blue pens and 1 black pen?

- a.  $15/248$
- b.  $14/584$
- c.  $17/582$
- d.  $16/243$

**Explanation:**

- Here, total number of pens = 9
- Probability of drawing 1 blue pen =  $4/9$
- Probability of drawing another blue pen =  $4/9$
- Probability of drawing 1 black pen =  $3/9$

→ Probability of drawing 2 blue pens and 1 black pen =  $\frac{4}{9} * \frac{4}{9} * \frac{3}{9} = \frac{48}{729} = \frac{16}{243}$

**Question 22:** If  $P(A) = 0.18$ ,  $P(B) = 0.5$  and  $P(B|A) = 0.2$ , find  $P(A \cap B)$ ?

- a. 0.32
- b. 0.36
- c. 0.16
- d. 0.64

**Explanation:**

- $P(B|A) = P(A \cap B)/P(A)$
- $P(A \cap B) = P(B|A) \times P(A)$
- $P(A \cap B) = 0.2 \times 0.18$
- $P(A \cap B) = 0.36$

**Question 23:** An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

- a.  $\frac{1}{7}$
- b.  $\frac{2}{7}$
- c.  $\frac{7}{3}$
- d.  $\frac{3}{7}$

**Explanation:**

- Let E and F denote respectively the events that first and second ball drawn are black.
- We have to find  $P(E \cap F)$  or  $P(EF)$ .
- Now  $P(E) = P(\text{black ball in first draw}) = \frac{10}{15}$
- Also given that the first ball drawn is black, i.e., event E has occurred, now there are 9 black balls and five white balls left in the urn. Therefore, the probability that the the second ball drawn is black, given that the ball in the first draw is black, is nothing but the conditional probability of F given that E has occurred.
- That is  $P(F|E) = \frac{9}{14}$
- By multiplication rule of probability, we have  
 $P(E \cap F) = P(E) P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$

**Question 24:** Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is:

- a.  $3/20$
- b.  $29/34$
- c.  $47/100$
- d.  $13/102$

**Explanation:**

- Let S be the sample space.
- Then,  $n(S) = {}^{52}C_2 = (52 \times 51)/(2 \times 1) = 1326$ .
- Let E = event of getting 1 spade and 1 heart.
- $n(E)$  = number of ways of choosing 1 spade out of 13 and 1 heart out of 13 =  ${}^{13}C_1 \times {}^{13}C_1 = 169$ .
- $P(E) = n(E)/n(S) = 169/1326 = 13/102$ .

**Question 25:** A speaks truth in 75% of cases and B in 80% of cases. In what percentage of cases are they likely to contradict each other, narrating the same incident:

- a.  $30/100$
- b.  $35/100$
- c.  $45/100$
- d.  $50/100$

**Explanation:**

- Let A = Event that A speaks the truth
- B = Event that B speaks the truth
- Then  $P(A) = 75/100 = \frac{3}{4}$
- $P(B) = 80/100 = \frac{4}{5}$
- $P(A\text{-lie}) = 1 - \frac{3}{4} = \frac{1}{4}$
- $P(B\text{-lie}) = 1 - \frac{4}{5} = \frac{1}{5}$
- Now, A and B contradict each other = [A lies and B true] or [B true and B lies]  
=  $P(A).P(B\text{-lie}) + P(A\text{-lie}).P(B)$   
=  $(\frac{3}{4} \times \frac{1}{5}) + (\frac{1}{4} \times \frac{4}{5}) = \frac{7}{20}$   
=  $(\frac{7}{20} \times 100) = 35\%$

### 8.6.1: ASSIGNMENT - PROBABILITY

1. Two coins are tossed simultaneously. What is the probability of getting only one head?

- a.  $\frac{1}{4}$
- b.  $\frac{1}{2}$
- c.  $\frac{1}{3}$
- d. none of these

2. Three coins are tossed together. What is the probability of getting exactly two heads?

- a.  $\frac{1}{2}$
- b.  $\frac{3}{8}$
- c.  $\frac{5}{8}$
- d.  $\frac{4}{7}$

3. A coin is tossed thrice. What is the probability of getting at most two tails?

- a.  $\frac{3}{8}$
- b.  $\frac{5}{9}$
- c.  $\frac{5}{8}$
- d.  $\frac{7}{8}$

4. Two dice are rolled together. What is the probability of getting a doublet?

- a.  $\frac{1}{6}$
- b.  $\frac{2}{5}$
- c.  $\frac{1}{36}$
- d.  $\frac{5}{36}$

5. Two dice are rolled together. What is the probability that the sum is neither 7 nor 11?

- a.  $\frac{7}{8}$
- b.  $\frac{7}{9}$
- c.  $\frac{1}{36}$
- d.  $\frac{11}{36}$

6. A card is drawn from a pack of 52 cards. What is the probability that it is a diamond king?

- a.  $\frac{1}{13}$
- b.  $\frac{1}{52}$
- c.  $\frac{3}{28}$
- d.  $\frac{1}{26}$

**7.** Two cards are drawn at random from a pack of 52 cards. What is the probability that they are king and queen?

- |            |              |
|------------|--------------|
| a. $1/26$  | b. $4/125$   |
| c. $8/663$ | d. $125/999$ |

**8.** Four cards are drawn at random from a pack of cards. What is the probability that they are from each suit?

- |                 |                 |
|-----------------|-----------------|
| a. $4/13$       | b. $325/936$    |
| c. $2197/20825$ | d. $2134/56981$ |

**9.** What is the probability that a non-leap year will have 53 Sundays?

- |             |            |
|-------------|------------|
| a. $53/365$ | b. $1/365$ |
| c. $52/365$ | d. $1/7$   |

**10.** If the letters of the word 'STATISTICS' are arranged at random, what is the probability that three T's always come together?

- |           |           |
|-----------|-----------|
| a. $1/15$ | b. $2/23$ |
| c. $1/10$ | d. $3/20$ |

**11.** Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Find the probability that exactly two of them will be children.

- |            |            |
|------------|------------|
| a. $2/9$   | b. $3/16$  |
| c. $10/21$ | d. $11/24$ |

**12.** A set of four books is arranged on a shelf. What is the probability that they will be in correct order?

- a.  $\frac{1}{2}$
- b.  $\frac{1}{3}$
- c.  $\frac{1}{24}$
- d. cannot be determined

**13.** There are 36 books numbered from 1 to 36. A book is selected at random. What is the probability that the number on the book is either divisible by 3 or a perfect square?

- a.  $\frac{1}{18}$
- b.  $\frac{4}{9}$
- c.  $\frac{2}{9}$
- d. none of these

**14.** There are 200 bicycles numbered from 1 to 200. A bicycle is chosen at random. What is the probability that the number on the bicycle is either divisible by 2 or 3 or 5?

- a.  $\frac{3}{20}$
- b.  $\frac{25}{90}$
- c.  $\frac{73}{100}$
- d.  $\frac{146}{100}$

**15.** There are 6 positive and 8 negative numbers. Four numbers are chosen at random and multiplied. What is the probability that their product will always be positive?

- a.  $\frac{505}{1001}$
- b.  $\frac{1}{2}$
- c.  $\frac{102}{305}$
- d.  $\frac{607}{1000}$

**16.** In a painting competition, there are 50 participants. Out of them, 15 people decided to make nature paintings, 10 people decided to make clay paintings and the remaining decided to make portraits. What is the probability of winning a nature painting or a clay painting or a portrait?

- a. 0
- b.  $\frac{1}{2}$
- c. 1
- d. none of these

**17.** A and B are two independent events. The probability that both A and B occur is  $\frac{1}{6}$  and the probability that neither of them occurs is  $\frac{1}{3}$ . Find the probability of occurrence of A.

- a.  $\frac{1}{2}$
- b.  $\frac{1}{3}$
- c.  $\frac{1}{4}$
- d. cannot be determined

**18.** The chance that it will rain in a city is 50%. What is the chance that it will rain 3 out of 5 days?

- a.  $\frac{3}{5}$
- b.  $\frac{1}{5}$
- c.  $\frac{5}{16}$
- d.  $\frac{7}{9}$

**19.** The chance that a bus comes on time is 0.40. What is the chance that the bus will not come on time 3 out of 5 days?

- a.  $\frac{144}{625}$
- b.  $\frac{216}{625}$
- c.  $\frac{221}{223}$
- d.  $\frac{215}{311}$

**20.** There are three locks and their keys. A person tries to match the locks and their keys without any prior knowledge. What is the probability of two correct matching?

- a.  $\frac{1}{9}$
- b.  $\frac{1}{2}$
- c.  $\frac{1}{3}$
- d. none of these

**21.** Out of 800 families with 5 children each, how many would you expect to have 3 boys? (There are equal chances of occurrence for a boy and a girl.)

- a. 322
- b. 634
- c. 250
- d. 410



**22.** The probability of student A passing an examination is  $\frac{2}{5}$  and of student B passing is  $\frac{4}{7}$ . Assuming the two events “A passes” and “B passes” as independent, find the probability of only A passing the examination.

- a.  $\frac{6}{35}$
- b.  $\frac{1}{5}$
- c.  $\frac{8}{35}$
- d.  $\frac{9}{35}$

**23.** X, Y and Z in order toss an unbiased coin. The one who first throws head wins the game. Find their respective chances of winning the game. Assume that the game may continue indefinitely.

- a.  $\frac{4}{7}, \frac{2}{7}, \frac{1}{7}$
- b.  $\frac{4}{7}, \frac{1.8}{7}, \frac{1.2}{7}$
- c.  $\frac{3}{7}, \frac{2}{7}, \frac{2}{7}$
- d.  $\frac{5}{7}, \frac{0.6}{7}, \frac{0.4}{7}$

**24.** A 4-digit number is formed from the digits 5, 6, 7, and 8 with no repetition of digits. What is the probability that the number is divisible by 5?

- a.  $\frac{3}{4}$
- b.  $\frac{1}{2}$
- c.  $\frac{1}{4}$
- d.  $\frac{2}{5}$

**25.** The odds against a certain event A are 4:3 and the odds in favor of another independent event B are 5:4. What is the probability that at least one of the events will occur?

- a.  $\frac{46}{63}$
- b.  $\frac{44}{63}$
- c.  $\frac{50}{63}$
- d.  $\frac{47}{63}$

### ***8.6.1 Answer Keys***

<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>	<b>Q. No</b>	<b>Ans</b>
<b>1</b>	b	<b>6</b>	b	<b>11</b>	c	<b>16</b>	c	<b>21</b>	b
<b>2</b>	b	<b>7</b>	c	<b>12</b>	c	<b>17</b>	d	<b>22</b>	a
<b>3</b>	d	<b>8</b>	c	<b>13</b>	b	<b>18</b>	c	<b>23</b>	a
<b>4</b>	a	<b>9</b>	d	<b>14</b>	c	<b>19</b>	b	<b>24</b>	c
<b>5</b>	b	<b>10</b>	a	<b>15</b>	a	<b>20</b>	d	<b>25</b>	d

